New Keynesian Monetary Policy Analysis

for Central Bank Economists

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Objectives

- Much of recent work on monetary policy within the New Keynesian framework
- Objectives of monetary policy theory
  - Positive: To prove an understanding of links between monetary policy and the aggregate economy, and an account of the monetary policy transmission mechanism
  - Normative: To identify the objectives of monetary policy and provide prescriptions for how it should be conducted to obtain those objectives
- Objectives of this course are to give participants
  - A sufficient knowledge of the basic New Keynesian monetary model to read quickly and understand additions to the literature
  - An idea if not an overview of the issues dealt with
  - A background knowledge of the modelling approach behind the new DSGE model under development at the Central Bank of Iceland

Readings

- **Jordi Galí (2008). Monetary Policy, Inflation, and the Business Cycle – An Introduction to the New Keynesian Framework. Princeton University Press**
- Carl Walsh (2003). *Monetary Theory and Policy*, 2nd edition. MIT Press
- Michael Woodford (2003). *Interest and Prices - Foundations of a Theory of Monetary Policy.* Princeton University Press
- David N. DeJong with Chatan Dave (2007). *Structural Macroeconometrics.* Princeton University Press

Outline

1. A Basic New Keynesian Model
   Tuesday 9:00-10:20: Chapters 1-3
2. Monetary Policy, Inflation and the Business Cycle
   Tuesday 10:40-12:00: Chapters 3
3. The Design of Monetary Policy
   Wednesday 9:00-10:20: Chapters 4-5
4. The Small Open Economy
   Wednesday 10:40-12:00: Chapter 7 (and 8)

Lecture 1:

A Basic New Keynesian Model

A very brief history of macroeconomics (1 of 3)

- Classicals: Emphasis on self-correcting forces in the economy
- Macroeconomics born with the Great Depression and Keynes (1936)
    - Formalised in IS-LM/AD-AS models
    - Behavioural relations specified directly, e.g. "the psychology of the community is such that when aggregate real income is increased agggregate consumption is increased, but not by so much as income":

$$C = c_0 + c_1 Y^D$$

    - Backward-looking expectations and sticky prices
    - Fiscal and monetary policy can be used to stabilise the economy
- Neoclassical Synthesis: Keynesian short run and classical long run
- Traditional macroeconometric models: Estimate relations equation by equation and combine for policy analysis

A very brief history of macroeconomics (2 of 3)

- Neoclassical critique
  - Sargent and Wallace JPE 1975: Economic policy is ineffective in Keynesian models when expectations are rational
  - Lucas CRCPP 1976: Traditional macroeconomic models are useless for policy evaluation since estimated parameters are not invariant to policy
- Real business cycle (RBC) literature beginning with Kydland and Prescott EC 1982 introduces the DSGE methodology
  - Dynamic: Infinitely-lived, forward-looking agents
  - Stochastic: Exogenous shocks with a known distribution
  - General Equilibrium: Well-defined decision problems solved by all agents taking all interactions into account
  - First-order conditions instead of *ad hoc* behavioural relations
- Assumptions: Perfect competition and flexible prices
  - Business cycles are efficient fluctuations driven by productivity shocks
  - No role for monetary policy

A very brief history of macroeconomics (3 of 3)

- Early New Keynesian literature (Mankiw and Romer, eds., 1991): Microfoundations for Keynesian features in partial-equilibrium, deterministic or static models
- Current New Keynesian literature combines the DSGE approach with a Keynesian role for stabilisation policies due to
  - Monopolistic competition (output below potential)
  - Nominal rigidities (monetary effects)
  - Other frictions
- New Keynesian Economics aka the *New Neoclassical Synthesis*
- Macroeconometric state of the art ("DSGE modelling") is to estimate/calibrate "deep" parameters jointly

The basic New Keynesian model

- Agents: A large number of rational, optimising households and firms, a central bank, and a government
- Commodities: Consumption goods, leisure, labour, money and bonds (no capital for simplicity)
- Preferences: Household gain utility from consumption, leisure and money services
- Technology: Production technology and a price-setting mechanism
- Information: Shock realised and observed at the beginning of period (time is discrete), then markets open
- Object of study is the decentralised equilibrium: Stochastic process for endogenous variables consistent with the optimal behaviour of agents given objectives and constraints as well as the clearing of all markets

Households

Representative household's decision problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

s.t.

$$P_t C_t + Q_t B_t + M_t = B_{t-1} + M_{t-1} + W_t N_t - T_t$$

- Utility of consumption and disutility of work
- Real balances in the utility function
- $Q_t \equiv (1 + \mathcal{I}_t)^{-1}$ where $\mathcal{I}_t$ is the risk-free nominal interest rate
- Solvency condition implicit: $\lim_{T\to\infty} E_t \beta^T (B_T + M_T) \geq 0$

Households

Expectations a probability-weighted sum

$$\begin{aligned}
& \max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\left(M_t / P_t\right)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \\
= \quad & \max \sum_{t=0}^{\infty} \sum_{\varsigma^t} \xi_{0,t}\left(\varsigma^t\right) \beta^t U \left(C_t\left(\varsigma^t\right), \frac{M_t\left(\varsigma^t\right)}{P_t\left(\varsigma^t\right)}, N_t\left(\varsigma^t\right) \right)
\end{aligned}$$

- ς_t is an *event* (the realisation of shocks in period t)
- ς^t is the *state of nature* in period t (the history of events)

$$\varsigma^t = \{\varsigma_0, \varsigma_1, \ldots, \varsigma_t\}$$

- $\xi_{s,t+1}\left(\varsigma^{t+1}\right)$ is the probability of ς^{t+1} conditional on ς^s

Households

Lagrangian (suppressing dependence on state)

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty}\sum_{\varsigma^t}\xi_{0,t}\beta^t\left\{\left[\frac{C_t^{1-\sigma}}{1-\sigma}+\frac{(M_t/P_t)^{1-\nu}}{1-\nu}-\frac{N_t^{1+\varphi}}{1+\varphi}\right]\right.\\ &\qquad \left.-\lambda_t\left[P_tC_t+Q_tB_t+M_t-B_{t-1}-M_{t-1}-W_tN_t+T_t\right]\right\}\end{aligned}$$

- F.o.c. for consumption at time t (and state ς^t):

$$\frac{\partial\mathcal{L}}{\partial C_t}=C_t^{-\sigma}-\lambda_tP_t=0$$

- F.o.c. for bonds at time t (and state ς^t):

$$\frac{\partial\mathcal{L}}{\partial B_t}=-\xi_{0,t}\lambda_tQ_t+\beta\sum_{\varsigma^{t+1}}\xi_{0,t+1}\lambda_{t+1}=0$$

Households

- Combine f.o.c. for consumption in two periods/states of nature:

$$\left.\begin{array}{l} C_t^{-\sigma} = \lambda_t P_t \\ C_{t+1}^{-\sigma} = \lambda_{t+1} P_{t+1} \end{array}\right\} \Rightarrow \frac{\lambda_{t+1}}{\lambda_t} = \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}}$$

- Combine this with f.o.c. for bonds

$$\xi_{0,t} \lambda_t Q_t = \beta \sum_{\varsigma^{t+1}} \xi_{0,t+1} \lambda_{t+1}$$

$$\Leftrightarrow \quad Q_t = \beta \sum_{\varsigma^{t+1}} \frac{\xi_{0,t+1}}{\xi_{0,t}} \frac{\lambda_{t+1}}{\lambda_t} = \beta \sum_{\varsigma^{t+1}} \xi_{t,t+1} \frac{\lambda_{t+1}}{\lambda_t}$$

to get Euler equation

$$Q_t = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right]$$

Households

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \sum_{\varsigma^t} \xi_{0,t} \left\{ \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \right.$$
$$\left. -\lambda_t \left[P_t C_t + Q_t B_t + M_t - B_{t-1} - M_{t-1} - W_t N_t + T_t \right] \right\}$$

- F.o.c. for labour:

$$\frac{\partial \mathcal{L}}{\partial N_t} = -N_t^{\varphi} - \lambda_t W_t = 0$$

Since $C_t^{-\sigma} = \lambda_t P_t$ this gives the labour supply relation

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{C_t^{-\sigma}}$$

Households

Lagrangian

$$\begin{aligned}\mathcal{L} \;=\; & \sum_{t=0}^{\infty}\sum_{\varsigma^t}\xi_{0,t}\left\{\beta^t\left[\frac{C_t^{1-\sigma}}{1-\sigma}+\frac{(M_t/P_t)^{1-\nu}}{1-\nu}-\frac{N_t^{1+\varphi}}{1+\varphi}\right]\right.\\ & \left.-\lambda_t\left[P_tC_t+Q_tB_t+M_t-B_{t-1}-M_{t-1}-W_tN_t+T_t\right]\right\}\end{aligned}$$

- F.o.c. for money:

$$\frac{\partial\mathcal{L}}{\partial M_t}=\frac{\xi_{0,t}}{P_t}\left(\frac{M_t}{P_t}\right)^{-\nu}-\lambda_t+\beta\sum_{\varsigma^{s+1}}\xi_{0,t+1}\lambda_{t+1}=0$$

$$\Leftrightarrow\quad\left(\frac{M_t}{P_t}\right)^{-\nu}=P_t\lambda_t\left[1-\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\right]$$

Combine with f.o.c. for bonds and consumption to get money demand

$$\frac{M_t}{P_t}=C_t^{\frac{\sigma}{\nu}}\left[1-Q_t\right]^{-\frac{1}{\nu}}$$

Households: Summary

- Euler equation

$$Q_t = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right]$$

- Labour supply

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{C_t^{-\sigma}}$$

- Money demand

$$\frac{M_t}{P_t} = C_t^{\frac{\sigma}{\nu}} \left[1 - Q_t\right]^{\frac{1}{\nu}}$$

Monopolistic competition

- Problem: We need to give firms market power to model price setting
- Trick (Blanchard and Kiyotaki AER 1987): Use the Dixit and Stiglitz AER 1977 formalisation of monopolistic competition
  - Monopolistic competition (Chamberlin, 1933): Market structure where a 'large' number of 'small' firms supply goods that are imperfect subsitutes to other goods
- Let consumption be an index of a continuum of goods each produced by an individual firm indexed on the unit interval $i \in [0, 1]$:

$$C_t = \left(\int_0^1 C_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- Define a price index over the continuum of goods:

$$P_t = \left(\int_0^1 P_t\left(i\right)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

Monopolistic competition

This adds an *intratemporal* decision problem to the household *intertemporal* decision (above):

$$\min_{c_t(i)} \int_0^1 P_t(i)\, C_t(i)\, di \text{ s.t. } \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di \right)^{\frac{\varepsilon}{\varepsilon-1}} = C_t$$

- From f.o.c. we get demand for good *i*:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$$

- Measure of market power: $\varepsilon > 1$ is the price elasticity of demand
- Perfect competition special case: $\varepsilon \longrightarrow \infty$
- Consistency with intertemporal problem since

$$\int_0^1 P_t(i)\, C_t(i)\, di = P_t C_t$$

Price-setting

- Monopolistic competition means price setting
- Flexible prices: Firms are free to set new prices each period
- Sticky prices : Firms are not free to set prices each period (or price-setting is costly)
- Simplest sticky-price model: Set price one period in advance
- Time-dependent – staggered price setting
    - Fischer JPE 1977 (predetermined for fixed period)
    - Taylor AER 1979 (fixed for fixed period)
    - Calvo JME 1983 (fixed for random period)
- Price-adjustment costs
    - Rotemberg RES 1982
- State-dependent price setting:
    - Barro RES 1972/Mankiw QJE 1985 (menu costs in PE)
    - Blanchard and Kiyotaki AER 1987 (menu costs in GE)

Price-setting

- Sticky prices in DSGE
  - Calvo staggering (Yun JME 1996)
  - Taylor staggering (Chari, Kehoe and McGratten EC 2000)
  - Menu costs (Gertler and Leahy JPE 2008)
- Here staggering because (Chari, Kehoe and McGratten, EC 2000):

> '*Staggered price-setting provides a promising mechanism to generate long periods of endogenous price stickiness with the small friction of short periods of exogenous price stickiness*'

- Calvo contracts because easy to handle
- ...but Rotemberg is likely to take over in future

Price-setting

- Staggering as in Calvo JME 1983: Random duration of prices
- Each period, each firm is allowed to reset its price with a fixed probability $(1-\theta)$
- Hence a fraction $(1-\theta)$ of firms reset prices each period
- θ: measure of price rigidity
    - Expected time between price changes: $1/\left(1-\theta\right)$
    - Complete price rigidity: $\theta = 1$
    - Full flexibility: $\theta = 0$
- All firms setting a new price in period t set the same price P_t^*
- Law of motion

$$P_t = \left[\int_0^1 P_t\left(i\right)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} = \left[\theta\left(P_{t-1}\right)^{1-\varepsilon} + \left(1-\theta\right)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Price-setting

- Suppose all firms initially charge $P = 100$ but a shock leads to a one-period hike in inflation from 0% to 5%
- Assume Calvo price-setting where 50% of firms can change prices each period
- Price dynamics (simplified price index):

Period	Price
0	$100\% \cdot 100 = 100$
1	$50\% \cdot 100 + 50\% \cdot 110 = 105$
2	$50\% \cdot (50\% \cdot 100 + 50\% \cdot 110) + 50\% \cdot 105$
	$= 25\% \cdot 100 + 25\% \cdot 110 + 50\% \cdot 105 = 105$
3	$50\% \cdot (25\% \cdot 100 + 25\% \cdot 110 + 50\% \cdot 105) + 50\% \cdot 105$
	$= 12.5\% \cdot 100 + 12.5\% \cdot 110 + 75\% \cdot 105 = 105$
$\vdots$	$\vdots$
∞	$100\% \cdot 105 = 105$

Firms

- A continuum of firms $i \in [0,1]$ each producing one of the goods according to
$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$
where A_t is an exogenous technology process
- Nominal total costs
$$NTC_t(i) = W_t N_t(i) = \tilde{W}_t \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}}$$
- Nominal marginal costs
$$\begin{aligned} NMC_t(i) &= \frac{\partial NTC_t(i)}{\partial Y_t(i)} = \frac{W_t}{(1-\alpha) A_t} \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}-1} \\ &= \frac{W_t}{(1-\alpha) A_t N_t(i)^{-\alpha}} = \frac{W_t}{MPN_t(i)} \end{aligned}$$

Firms

- A firm allowed to reset its price at time t does so to solve

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[P_t^* Y_{t+k|t} - W_t N_{t+k|t}\right]$$
$$\text{s.t.} \quad Y_{t+k|t} = A_t N_{t+k|t}^{1-\alpha}$$
$$\text{and} \quad Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$$

- $Q_{t,t+k}$ is the stochastic discount factor:

$$\begin{aligned} Q_{t,t+k} &= \beta^k \frac{\lambda_{t+k}}{\lambda_t} \\ E_t Q_{t,t+1} &= Q_t = \frac{1}{1+\mathcal{I}_t} \end{aligned}$$

- θ^k discounts probability that price will have been changed in period k (maximisation over "states" where price will be in place)

Firms

- By subsitution, the f.o.c can be found to be

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[P_t^* \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k} - W_t \left(\frac{\left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k}}{A_t} \right)^{\frac{1}{1-\alpha}} \right]$$

- Differentiation w.r.t. P_t^* gives

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[(1-\varepsilon) Y_{t+k|t} + \frac{\varepsilon W_t Y_{t+k|t}}{P_t^* (1-\alpha) A_t} \left(\frac{Y_{t+k|t}}{A_t} \right)^{\frac{1}{1-\alpha}-1} \right]$$

- Setting this to zero gives f.o.c. (after straightforward manipulations):

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[P_t^* - \frac{\varepsilon}{\varepsilon - 1} NMC_{t+k|t} \right] = 0$$

Firms

- Price-setting first-order condition

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[P_t^* - \frac{\varepsilon}{\varepsilon - 1} NMC_{t+k|t} \right] = 0$$

- Price set as mark-up over marginal costs in discounted expected terms
- If $\theta = 0$ (flexible prices), f.o.c. collapses to

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} NMC_{t+k|t}$$

- If $\theta = 0$ *and* $\tau = \varepsilon^{-1}$ we get

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\left(1 - \varepsilon^{-1}\right) W_t}{\left(1 - \alpha\right) A_t N_t \left(i\right)^{-\alpha}} = NMC_{t+k|t}$$

- In steady state we get

$$P = \frac{\varepsilon}{\varepsilon - 1} NMC \Leftrightarrow MC = \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{-1}$$

Firms: Summary

- Production

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

- Price setting

$$0 = E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[P_t^* - \frac{\varepsilon}{\varepsilon - 1} NMC_{t+k|t} \right]$$

$$NMC_t(i) = \frac{W_t}{(1-\alpha) A_t N_t(i)^{-\alpha}}$$

$$P_t = \left[\theta (P_{t-1})^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

Log-linearisation

- Model consists of the household and firm equations above, a specification of monetary policy (later) and the market clearing conditions

$$\begin{aligned} Y_t\left(i\right) &= C_t\left(i\right) \Rightarrow Y_t = C_t \\ N_t &= \int_0^1 N_t\left(i\right) di \end{aligned}$$

- Problem: This is a complicated system of non-linear expectational equivations that we cannot solve
- Solution: Work with log-linear approximation around steady state
- Cost: Model only accurate 'close' to the steady state (i.e. bounds on shocks we can consider)

Log-linearisation

- For any variable X_t with steady state value X, we use the notation

$$\begin{aligned} x_t &\equiv \ln(X_t) \\ \hat{x}_t &= x_t - x \approx \frac{X_t - X}{X} \end{aligned}$$

- If variables X_t and Z_t be related by some function $f(.)$:

$$X_t = f(Z_t)$$

- Then this relation can be log-linearised by first taking logs

$$x_t = \ln[f(\exp z_t)] = \ln[g(z_t)]$$

and then by taking a first-order Taylor approximation

$$x_t \approx \ln[g(z)] + \frac{\partial g(z)}{\partial z}\frac{z}{g(z)}(z_t - z)$$

$$\iff \quad \hat{x}_t \approx \eta_{xz}\hat{z}_t$$

Log-linearisation

- The labour supply relation is easy

$$\begin{aligned} \frac{W_t}{P_t} &= \frac{N_t^{\varphi}}{C_t^{-\sigma}} \\ \Rightarrow \quad w_t - p_t &= \sigma c_t + \varphi n_t \end{aligned}$$

Log-linearisation

- The money demand relation is more difficult

$$\frac{M_t}{P_t} = C_t^{\frac{\sigma}{\nu}} \left[1 - Q_t\right]^{\frac{1}{\nu}} = C_t^{\frac{\sigma}{\nu}} \left[1 - \exp\left(-i_t\right)\right]^{\frac{1}{\nu}}$$

$$\text{where } i = -\ln Q_t = \ln\left(1 + \mathcal{I}_t\right) \approx \mathcal{I}_t$$

$$\Rightarrow \quad m_t - p_t = \frac{\sigma}{\nu} c_t - \frac{1}{\nu} \ln\left[1 - \exp\left(-i_t\right)\right]$$

- Take first-order Taylor

$$\ln\left[1 - \exp\left(-i_t\right)\right] \approx \ln\left[1 - \exp\left(-i\right)\right] + \frac{\exp\left(-i\right)}{1 - \exp\left(-i\right)} \frac{1}{\nu} \left(i_t - i\right)$$

- Insert this to get

$$m_t - p_t \approx \frac{\sigma}{\nu} c_t - \eta i_t$$

where $\eta = \frac{1}{\nu} \frac{\exp(-i)}{1 - \exp(-i)}$ which holds "up to an unimportant constant" (or replace with deviations from the steady state)

Log-linearisation

- Similarly with the Euler equation

$$1 = \beta E_t \left[\frac{1}{Q_t} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] = E_t \left[\exp \left(i - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho \right) \right]$$

where $\rho = -\ln \beta$ and $\pi_t = p_t - p_{t-1}$

- Note $i = \sigma \Delta c - \pi - \rho$
- Take 1st order Taylor approximation of term in brackets

$$\begin{aligned} & \exp \left(i - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho \right) \\ \approx \quad & e^0 + e^0 \left(i_t - i \right) - e^0 \left(\Delta c_{t+1} - \Delta c \right) - e^0 \left(\pi_{t+1} - \pi \right) \\ = \quad & 1 + i_t - \Delta c_{t+1} - \pi_{t+1} - \rho \end{aligned}$$

- Inserting this in Euler equation gives

$$c_t \approx E_t \left(c_{t+1} \right) - \sigma^{-1} \left[i_t - E_t \left(\pi_{t+1} \right) - \rho \right]$$

Log-linearisation

- And it gets worse: Production levels will be different across firms due to the price setting mechanism, but we look for a relation for aggregate production
- Substituting the individual production and demand functions into the labour market clearing condition gives

$$N_t = \int_0^1 N_t(i)\, di = \int_0^1 \left(\frac{1}{A_t} \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \right)^{1-\alpha} di$$

$$\Rightarrow \quad (1-\alpha)\, n_t = y_t - a_t + d_t$$

- The price dispersion measure $d_t = \int_0^1 \left(P_t(i) \,/\, P_t \right)^{\frac{\varepsilon}{\alpha-1}} di$ can be shown to be of 'second order'. Hence, we have to a first-order approximation

$$y_t \approx a_t + (1-\alpha)\, n_t$$

Log-linearisation

- Price setting

$$0 = E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[P_t^* - \frac{\varepsilon}{\varepsilon - 1} NMC_{t+k|t} \right]$$

$$\Rightarrow \quad p_t^* - p_t \approx (1 - \theta\beta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[\widehat{mc}_{t+k|t} + p_{t+k} \right]$$

- Law of motion of price index

$$P_t = \left[\theta \left(P_{t-1} \right)^{1-\varepsilon} + (1 - \theta) \left(P_t^* \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

$$\Rightarrow \quad p_t = \theta p_{t-1} + (1 - \theta) p_t^*$$

$$\Leftrightarrow \quad \pi_t \approx (1 - \theta) (p_t^* - p_t)$$

Log-linearisation

- Using tedious but trivial algebra, the price-setting f.o.c and the law of motion of the price index can be combined to give

$$\pi_t \approx \beta E_t \left(\pi_{t+1}\right) + \lambda \widehat{mc}_t$$

where

$$\begin{aligned}
\lambda &\equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \\
\widehat{mc}_t &\equiv mc_t - \ln\left(\frac{\varepsilon}{\varepsilon-1}\right) \equiv mc_t + \mu \\
mc_t &\equiv (w_t - p_t) - mpn_t = \widehat{mc}_{t+k|t} - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* - p_t\right)
\end{aligned}$$

Log-linearisation: Summary

- Euler

$$c_t = E_t\left(c_{t+1}\right) - \sigma^{-1}\left[i_t - E_t\left(\pi_{t+1}\right) - \rho\right]$$

- Labour supply

$$w_t - p_t = \sigma c_t + \varphi n_t$$

- Money demand

$$m_t - p_t = \frac{\sigma}{\nu} c_t - \eta i_t$$

- Production

$$y_t = a_t + \left(1 - \alpha\right) n_t$$

- Price setting

$$\begin{aligned} \pi_t &= \beta E_t\left(\pi_{t+1}\right) + \lambda \widehat{mc}_t \\ \widehat{mc}_t &= \left(w_t - p_t\right) - \left(a_t - \alpha n_t\right) - \ln\left(1 - \alpha\right) + \mu \end{aligned}$$

- Market clearing

$$y_t = c_t$$

Flexible price solution

- The solution in the special case with flexible prices ($\theta = 0$) turns out to be an important benchmark
- Strategy: Charaterise equilibrium dynamics in terms of flexible-price solution
- ...then solve and analyse the general model (next lecture)

Flexible-price solution

- Flexible prices imply

$$\lambda^{-1} = \frac{\theta}{(1-\theta)(1-\beta\theta)} \frac{1-\alpha+\alpha\varepsilon}{1-\alpha} = 0$$

- Hence

$$\begin{aligned} & \pi_t = E_t(\pi_{t+1}) + \lambda(mc_t + \mu) \\ \Leftrightarrow \quad & \lambda^{-1}[\pi_t - E_t(\pi_{t+1})] = mc_t + \mu = 0 \\ \Leftrightarrow \quad & mc_t = (w_t - p_t) - (a_t - \alpha n_t) - \ln(1-\alpha) = -\mu \\ \Leftrightarrow \quad & w_t - p_t = a_t - \alpha n_t - \ln(1-\alpha) + \mu \end{aligned}$$

- That is, we get a labour demand relation given by (up to a constant)

$$w_t - p_t = a_t - \alpha n_t = mpn_t$$

Flexible-price solution

- Model becomes

$$
\begin{aligned}
y_t &= E_t\left(y_{t+1}\right) - \sigma^{-1}\left[i_t - E_t\left(\pi_{t+1}\right) - \rho\right] \\
w_t - p_t &= \sigma y_t + \varphi n_t \\
m_t - p_t &= \frac{\sigma}{\nu} y_t - \eta i_t \\
y_t &= a_t + \left(1 - \alpha\right) n_t \\
w_t - p_t &= a_t - \alpha n_t
\end{aligned}
$$

Flexible-price solution

- Consider the three relations

$$\begin{aligned} w_t - p_t &= \sigma y_t + \varphi n_t \\ y_t &= a_t + (1-\alpha)\, n_t \\ w_t - p_t &= a_t - \alpha n_t \end{aligned}$$

- Combine labour supply and demand to get

$$\sigma y_t + \varphi n_t = a_t - \alpha n_t$$

- Insert production relation and rearrange to get solution for hours

$$\begin{aligned} & \sigma\left(a_t + (1-\alpha)\, n_t\right) + \varphi n_t = a_t - \alpha n_t \\ \Leftrightarrow \quad & n_t = \frac{1-\sigma}{\sigma(1-\alpha) + \varphi + \alpha} a_t \end{aligned}$$

Flexible-price solution

- This solution can be inserted in production function to get solution for output, and in labour demand relation to get solution for the real wage

$$\begin{aligned} y_t &= \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t \\ w_t - p_t &= \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t \end{aligned}$$

- From the Euler equation we get

$$\begin{aligned} r_t &\equiv i_t - E_t(\pi_{t+1}) = \rho + \sigma E_t(\Delta y_{t+1}) \\ &= \rho + \frac{\sigma(1+\varphi)}{\sigma(1-\alpha)+\varphi+\alpha} E_t(\Delta a_{t+1}) \end{aligned}$$

- Hence, the equilibrium dynamics of real variables determined independent of monetary policy specification and money demand

$$m_t - p_t = \frac{\sigma}{\nu} y_t - \eta i_t$$

Flexible-price solution

- Flexible-price solution in sum

$$\begin{aligned} n_t^n &= \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} a_t \\ y_t^n &= \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t \\ \omega_t^n &= \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t \\ r_t^n &= \rho + \frac{\sigma(1+\varphi)}{\sigma(1-\alpha)+\varphi+\alpha} E_t(\Delta a_{t+1}) \end{aligned}$$

- Superscript n refers to the 'natural' flexible price level of the variables
- This 'natural' equilibrium plays an important role in the interpretation of the general model with sticky prices

Equilibrium dynamics

- Price-setting relation under sticky prices

$$\begin{aligned}\pi_t &= \beta E_t(\pi_{t+1}) + \lambda(mc_t + \mu)\\ mc_t &= (w_t - p_t) - (a_t - \alpha n_t) - \ln(1-\alpha)\\ &= (\sigma y_t + \varphi n_t) - (a_t - \alpha n_t) - \ln(1-\alpha)\\ &= \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t - \frac{1+\varphi}{1-\alpha} a_t - \ln(1-\alpha)\end{aligned}$$

  where first the labour supply then production function has been used

- Under flexible price, we have

$$mc_t^n = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t^n - \frac{1+\varphi}{1-\alpha} a_t - \ln(1-\alpha) = -\mu$$

- Hence

$$mc_t + \mu = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)(y_t - y_t^n)$$

Equilibrium dynamics

- Combining gives the *New Keynesian Phillips curve*

$$\pi_t = \beta E_t\left(\pi_{t+1}\right) + \kappa \tilde{y}_t$$

where

$$\tilde{y}_t \equiv y_t - y_t^n$$

is the *output gap*, and

$$\kappa = \lambda\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) = \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$$

Equilibrium dynamics

- Model equations

$$\begin{array}{ll} & y_t = E_t y_{t+1} - \sigma^{-1}\left[i_t - E_t\left(\pi_{t+1}\right) - \rho\right] \\ \checkmark & w_t - p_t = \sigma y_t + \varphi n_t \\ & m_t - p_t = \frac{\sigma}{\nu} y_t - \eta i_t \\ \checkmark & y_t = a_t + \left(1 - \alpha\right) n_t \\ \checkmark & \pi_t = E_t\left(\pi_{t+1}\right) + \lambda\left(mc_t + \mu\right) \\ \checkmark & mc_t = \left(\tilde{w}_t - p_t\right) - \left(a_t - \alpha n_t\right) - \ln\left(1 - \alpha\right) \end{array}$$

Equilibrium dynamics

- Euler equation in general

$$y_t = E_t y_{t+1} - \sigma^{-1}\left[i_t - E_t\left(\pi_{t+1}\right) - \rho\right]$$

- Under flexible prices

$$y_t^n = E_t y_{t+1}^n - \sigma^{-1}\left[r_t^n - \rho\right]$$

- Combine to get the *dynamic IS equation*

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma^{-1}\left[i_t - E_t\left(\pi_{t+1}\right) - r_t^n\right]$$

Equilibrium dynamics

In sum

- New Keynesian Phillips curve:

$$\pi_t = \beta E_t \left\{ \pi_{t+1} \right\} + \kappa \tilde{y}_t$$

- Dynamic IS equation:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma^{-1} \left[i_t - E_t \left(\pi_{t+1} \right) - r_t^n \right]$$

- Money demand:

$$m_t - p_t = y_t - \eta i_t$$

Lecture 2:

Monetary Policy, Inflation and the Business Cycle

Equilibrium dynamics

In lecture one we derived a microfounded AS-AD/IS-LM model:

- New Keynesian Phillips curve (AS):

$$\pi_t = \beta E_t \left\{ \pi_{t+1} \right\} + \kappa \tilde{y}_t$$

- Dynamic IS equation:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma^{-1} \left[i_t - E_t \left(\pi_{t+1} \right) - r_t^n \right]$$

- Money demand (LM):

$$m_t - p_t = y_t - \eta i_t$$

- Close model by specification of monetary policy
  - Path of nominal supply of money → determine $\pi_t, \tilde{y}_t, i_t$
  - Interest rate rule → determine $\pi_t, \tilde{y}_t, m_t$

Equilibrium dynamics

- An interest rate rule more accurately describes actual central bank behaviour
- When monetary policy is specified as an interest rate rule, the LM relation becomes redundant (only needed for studying the development of monetary aggregates)
- Woodford (2003) therefore argues that the *cashless economy* can be used for monetary policy analysis
- Model here generates same dynamics of real variables as the cashless economy
  - If money affects the marginal utility of consumption, real variables are effected by monetary policy even when prices are flexible (monetary non-neutrality)

Equilibrium dynamics

- New Keynesian Phillips curve (AS):

$$\pi_t = \beta E_t\left(\pi_{t+1}\right) + \kappa \tilde{y}_t$$

- Dynamic IS equation:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma^{-1}\left[i_t - E_t\left(\pi_{t+1}\right) - r_t^n\right]$$

- Implications for monetary policy
  - Disturbances to system only in nominal interest rate (monetary policy) or the natural real rate of interest (real disturbances)
  - Relation between i_t and r_t^n determine inflation and output.
  - $i_t = r_t^n$ is the policy rate consistent with $\pi_t = 0$ for all t with the implication that $\tilde{y}_t = 0$ for all t
  - The natural interest rate is therefore also called the *neutral* interest rate

Equilibrium dynamics

- Woodford (2003, p. 238) terms this a neo-Wicksellian view of output/price determination (after Wicksell, 1898):

> *'Increases in output gaps and in inflation result from increases in the natural rate of interest that are not offset by a corresponding tightening of monetary policy...or alternatively from loosenings of monetary policy that are not justified by declines in the natural rate of interest'*

Equilibrium dynamics

- Ben Bernanke (Finance Committee luncheon of the Executives' Club of Chicago, 2005):

> *'...it is not helpful, in my view, to imagine the existence of some fixed target for the funds rate toward which policy should inexorably march. Instead, the correct procedure for setting policy requires the FOMC to continually update its forecast for the economy...on can search for indications of where the "neutral" funds rate is likely to be at a given point in time...the neutral rate is not a constant or a fixed objective but will change as the economy and economic forecasts evolve.'*

Equilibrium dynamics

- Alan Greenspan (answer to US Congress Joint Economic Committee, 2005):

> *′Although the concept of a "neutral interest rate" is a useful theoretical construct, difficulties in implementing it in practice limits its usefulness as a framework for monetary policymaking ...quantitative estimates of the level of such a rate are subject to considerable uncertainty... ...reliance on a single summary measure such as a neutral real interest rate would be unwise...′*

Equilibrium dynamics

- Phillips curve and dynamic IS equation:

$$\begin{aligned}\pi_t &= \beta E_t\left\{\pi_{t+1}\right\} + \kappa \tilde{y}_t \\ \tilde{y}_t &= E_t \tilde{y}_{t+1} - \sigma^{-1}\left[i_t - E_t\left(\pi_{t+1}\right) - r_t^n\right]\end{aligned}$$

- Iterating each of these equations forward gives

$$\begin{aligned}\pi_t &= \kappa \sum_{k=0}^{\infty} \beta^k E_t\left\{\tilde{y}_{t+k}\right\} = \lambda \sum_{k=0}^{\infty} \beta^k E_t\left\{\widehat{mc}_{t+k}\right\} \\ \tilde{y}_t &= -\frac{1}{\sigma} \sum_{k=0}^{\infty}\left(i_{t+k} - E_t\left(\pi_{t+k+1}\right) - r_{t+k}^n\right)\end{aligned}$$

- Implications
    - Inflation determined by expected future output gaps
    - *All* expected future short (real) interest rates matter for output and therefore for inflation

Equilibrium dynamics

- Central insight (Woodford, 2003, p. 2003):

> *'A central bank's primary impact on the economy comes about not through the level at which it sets current overnight interest rates, but rather though the way it affects private-sector expectations about the likely* **future** *path of overnight rates.'*

Model solution

- We need explicit specification of monetary policy to solve the model
- Example of interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

where $\phi_\pi, \phi_y > 0$ and

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

where ε_t^v is white noise

- Example of money growth rule

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

where ε_t^m is and exogenous shock

- Lecture 3 will address the desirability of alternative monetary policy rules

Model solution

- To get the solution when monetary policy follows the interest rate rule, insert this rule in the dynamic IS equation:

$$E_t\tilde{y}_{t+1} = \tilde{y}_t + \sigma^{-1}\left[\rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t - E_t\pi_{t+1} - r_t^n\right]$$

- Insert the New Keynesian Phillips

$$E_t\pi_{t+1} = \beta^{-1}\left[\pi_t - \kappa\tilde{y}_t\right]$$

for $E_t\pi_{t+1}$ to get a relation in $E_t\tilde{y}_{t+1}$, $\tilde{y}_t$ and π_t:

$$\begin{aligned} E_t\tilde{y}_{t+1} &= \tilde{y}_t + \sigma^{-1}\left[\rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t - \beta^{-1}\left[\pi_t - \kappa\tilde{y}_t\right] - r_t^n\right] \\ &= \left[1 + \sigma^{-1}\left(\phi_y + \frac{\kappa}{\beta}\right)\right]\tilde{y}_t + \sigma^{-1}\left(\phi_\pi - \beta^{-1}\right)\pi_t \\ & \quad -\sigma^{-1}\left(r_t^n - \rho - v_t\right) \end{aligned}$$

Model solution

- It follows that the model can be written as

$$\begin{bmatrix} E_t\tilde{y}_{t+1} \\ E_t\pi_{t+1} \end{bmatrix} = \mathbf{A}\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} + \mathbf{B}\left(r_t^n - \rho - v_t\right)$$

where

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} 1 + \sigma^{-1}\left(\phi_y + \frac{\kappa}{\beta}\right) & \sigma^{-1}\left(\phi_\pi - \beta^{-1}\right) \\ -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix} \\ \mathbf{B} &= \begin{bmatrix} -\sigma^{-1} \\ 0 \end{bmatrix} \end{aligned}$$

- This is a linear system of expectational difference equations
- Good news: We know how to solve this!

Model solution

- System in the form of Blanchard and Kahn EC 1980:

$$\begin{bmatrix} X_{t+1} \\ E_t Z_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} X_t \\ Z_t \end{bmatrix} + \mathbf{B} V_t \tag{1}$$

$$\begin{aligned} X_t &: \quad \text{endogenous and predetermined} \\ Z_t &: \quad \text{endogenous and non-predetermined} \\ V_t &: \quad \text{exogenous} \end{aligned}$$

- Rational expectations:

$$E_t P_{t+1} = E\left(P_{t+1} | \Omega_t\right)$$

$$\{X_{t-i}, Z_{t-i}, V_{t-i}\} \subseteq \Omega_t \ \forall i \geq 0$$

- Solution: A sequence of functions of variables in Ω_t satisfying (1) for all realizations.

Model solution

- Method begins with a Jordan decomposition of **A**:

  $$\mathbf{A} = \Lambda^{-1} J \Lambda$$

  where the diagonal elements of J are the eigenvalues of **A** in increasing absolute value moving from northwest to southeast
- The idea is to decouple the system in a stable part (eigenvalues within the unit circle) and an unstable part (eigenvalues outside the unit circle) such that nonpredetermined values depend on explosive eigenvalues:

  $$\begin{bmatrix} \tilde{X}_{t+1} \\ E_t \tilde{Z}_{t+1} \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} \tilde{X}_t \\ \tilde{Z}_t \end{bmatrix} + \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \mathbf{B} V_t$$

  where

  $$\begin{bmatrix} \tilde{X}_t \\ \tilde{Z}_t \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} X_t \\ Z_t \end{bmatrix}$$

Model solution

- Iterating the lower part forwards n times gives

$$\tilde{Z}_t = J_2^{-n} E_t \left(\tilde{Z}_{t+n} \right) - \sum_{i=0}^{n} J_2^{-(i+1)} E_t V_{t+i}$$

- Letting $n \longrightarrow \infty$ gives

$$\tilde{Z}_t = - \sum_{i=0}^{\infty} J_2^{-(i+1)} E_t V_{t+i}$$

  which we may calculate, e.g. given an assumed autoregressive stochastic process V_t
- This can now be mapped back into a solution for $Z_t = \Lambda_{22}^{-1} \left(\tilde{Z}_t - \Lambda_{21} X_t \right)$
- The solution for X_t now follows from the upper part of (1)

Model solution

- Solution derived assuming a unique solution, i.e. *determinacy*
- Blanchard and Kahn EC 1980 prove the following propositions
  1. If the number of eigenvalues of **A** outside the unit circle is equal to the number of non-predetermined variables, then there exists a unique solution (determinacy)
  2. If the number of eigenvalues outside the unit circle exceeds the number of non-predetermined variables, there is no solution
  3. If the number of eigenvalues outside the unit circle is less than the number of non-predetermined variables, there is an infinity of solutions (indeterminacy)

Model solution

- Numerical solution
  - Most common in the recent literature
  - Approach: Specify parameter values and implement on the computer
  - Motivation: DSGE models generally too complex to allow for analytical solutions
  - Allows for estimation of parameters
- Alternative: Solve analytically by hand by method of undetermined coefficients and 'inspect the mechanism' (Campbell JME 1994)
- The basic New Keynesian model is actually simple enough to be solved analytically

Monetary policy shock

- Solve by method of undetermined coefficients
- If only monetary policy shocks, guess a solution of the form

$$\begin{aligned}\tilde{y}_t &= \psi_{yv} v_t \Rightarrow E_t \tilde{y}_{t+1} = \psi_{yv} \rho v_t \\ \pi_t &= \psi_{\pi v} v_t \Rightarrow E_t \pi_{t+1} = \psi_{\pi v} \rho v_t\end{aligned}$$

- Inserting conjecture in Phillips curve gives

$$\psi_{\pi v} v_t = \beta^{-1} \left[\psi_{\pi v} v_t - \kappa \psi_{yv} v_t \right]$$

- Equate coefficients on v_t to get the restriction

$$\psi_{\pi v} = \beta^{-1} \left[\psi_{\pi v} - \kappa \psi_{yv} \right]$$

- Similarly from the dynamic IS equation

$$\psi_{yv} \rho = \left[1 + \sigma^{-1} \left(\phi_y + \frac{\kappa}{\beta} \right) \right] \psi_{yv} + \sigma^{-1} \left(\phi_\pi - \beta^{-1} \right) \psi_{\pi v} + \sigma^{-1}$$

- Hence two equations with the two unknowns $\psi_{\pi v}$ and ψ_{yv}

Monetary policy shock

- Solution

$$\begin{aligned}\tilde{y}_t &= \psi_{yv} v_t = -\frac{(1-\beta\rho_v)}{(1-\beta\rho_v)\left[\sigma(1-\rho_v)+\phi_y\right]+\kappa(\phi_\pi-\rho_v)} v_t \\ \pi_t &= \psi_{\pi v} v_t = -\frac{\kappa}{(1-\beta\rho_v)\left[\sigma(1-\rho_v)+\phi_y\right]+\kappa(\phi_\pi-\rho_v)} v_t\end{aligned}$$

- For $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ we have

$$\begin{aligned}\psi_{yv} &< 0 \\ \psi_{\pi v} &< 0\end{aligned}$$

- This condition will be given an economic interpretation in lecture 3
- Implication: An exogenous increase in the interest rate leads to a decline in the output gap and inflation

Monetary policy shock

- From the dynamic IS equation we get

$$\begin{aligned}
\hat{r}_t &= \frac{\sigma\left(1-\rho_v\right)\left(1-\beta\rho_v\right)}{\left(1-\beta\rho_v\right)\left[\sigma\left(1-\rho_v\right)+\phi_y\right]+\kappa\left(\phi_\pi-\rho_v\right)} v_t \\
\hat{\imath}_t &= \hat{r}_t + E_t\pi_{t+1} = \frac{\sigma\left(1-\rho_v\right)\left(1-\beta\rho_v\right)-\rho_v\kappa}{\left(1-\beta\rho_v\right)\left[\sigma\left(1-\rho_v\right)+\phi_y\right]+\kappa\left(\phi_\pi-\rho_v\right)} v_t
\end{aligned}$$

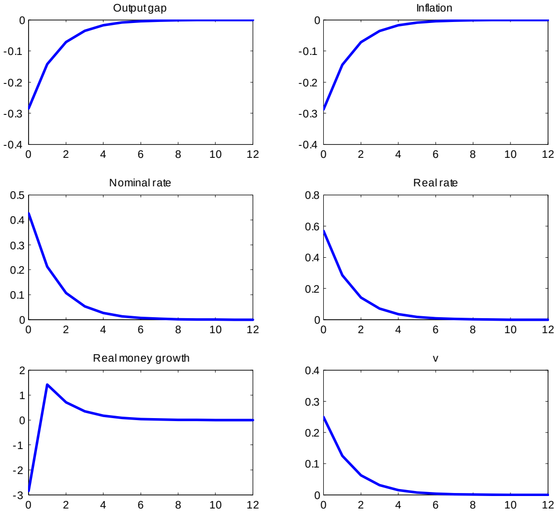
- Real interest rate increases unambiguously
- If the monetary policy shock is sufficiently persistent, the nominal rate will decline
- Can be shown from the LM relation that the change in the money supply supporting the equilibrium is of ambiguous sign, but positive when the nominal interest rate increases (liquidity effect)

Monetary policy shock

- The model solution can be used to quantify the effects of shocks given numerical values for the parameters
- We choose

$$\begin{aligned}
\beta &= 0.99 \\
\sigma &= \varphi = 1 \\
\alpha &= 1/3 \\
\varepsilon &= 6 \\
\theta &= 2/3 \\
\phi_\pi &= 1.5 \\
\phi_y &= 0.5/4 \\
\rho_v &= 0.5
\end{aligned}$$

Monetary policy shock



Technology shock

- Let monetary policy be given by

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t$$

where $\phi_\pi, \phi_y > 0$ and technology evolve according to

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

where ε_t^v is white noise

- Then by method of undetermined coefficients

$$\begin{aligned} \tilde{y}_t &= \frac{(1-\beta\rho_a)}{(1-\beta\rho_a)\left[\sigma(1-\rho_a)+\phi_y\right]+\kappa(\phi_\pi-\rho_a)}\hat{r}_t^n \\ \pi_t &= \frac{\kappa}{(1-\beta\rho_a)\left[\sigma(1-\rho_a)+\phi_y\right]+\kappa(\phi_\pi-\rho_a)}\hat{r}_t^n \end{aligned}$$

where

$$\hat{r}_t^n = -\sigma\frac{(1+\varphi)}{\sigma(1-\alpha)+\varphi+\alpha}(1-\rho_a)\,a_t$$

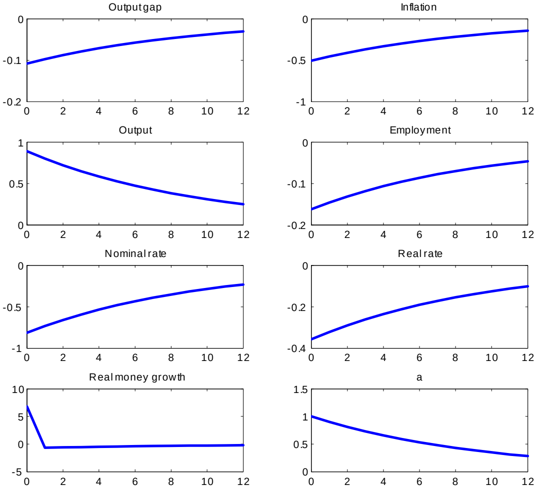
Technology shock

- From this we can derive expressions for output and hours as functions of technology

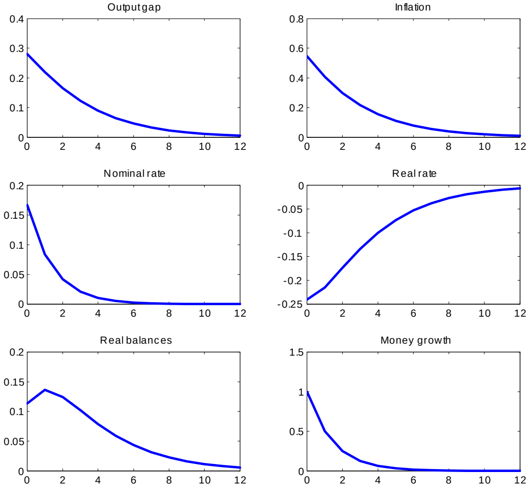
$$\begin{aligned} y_t &= y_t^n + \tilde{y}_t \\ n_t &= \frac{1}{1-\alpha}\left(y_t - a_t\right) \end{aligned}$$

- The Taylor principle sufficient to ensure a positive output response to a positive technology shock with $\rho_a \leq 1$
- The response of hours is ambiguous
- Calibration: $\rho_a = 0.9$

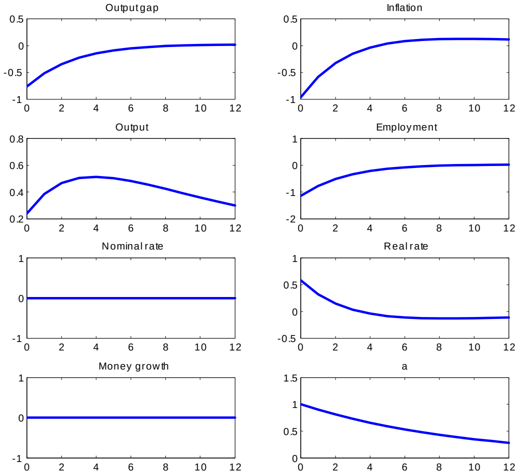
Technology shock



Exogenous money supply: Money supply shock



Exogenous money supply: Technology shock



Comparison of models

- The traditional expectations-augmented Phillips curve is given by

$$\pi_t = E_{t-1}\pi_t + \delta\hat{y}_t = \pi_{t-1} + \delta\hat{y}_t$$

where $\hat{y}_t$ is log deviation of GDP from trend.

  - Inflation determined by measure of economic activity
  - Different measures of $\hat{y}_t$
  - No direct monetary effects on inflation
  - Past inflation *matters*
  - Current inflation positively correlated with *past* output (output leads inflation)

Comparison of models

- The New Keynesian Phillips curve:

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa \hat{y}_t$$

where $\hat{y}_t \equiv y_t - y_t^n$

  - Inflation determined by measure of economic activity
  - $\hat{y}_t$ pinned down by theory (curve derived from first principles)
  - No direct monetary effects on inflation
  - Past inflation *irrelevant*
  - Inflation positively correlated with *future* output (inflation leads output)

Comparison of models

- Inflation according to the traditional Phillips curve is backward-looking
- Inflation in New Keynesian setting is forward-looking
- Intuition:

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}(\beta\theta)^k E_t\left(mc_{t+k|t} + p_{t+k}\right)$$

  - Inflation driven by price-setting firms
  - Price setting depends on current and expected future costs (unrelated to past inflation)

Comparison of models

- Which of the Phillips curves is consistent with the data?
- Estimations of hybrid Phillips curve (e.g., Fuhrer JMCB 1997):

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \{\pi_{t+1}\} + \delta \hat{y}_t$$

  - γ_b significantly different from zero, but not completely dominant
  - Not consistent with New Keynesian model
  - So theoretically appealing but empirically rejected?
- Critique: Results may be distorted by use of wrong output gap measure $\hat{y}_t$
  - Traditionally output gap defined as deviation from smooth trend (e.g. HP filter)
  - New-Keynesian output gap very different: Deviation from *virtual equilbrium* of flexible-price economy

Comparison of models

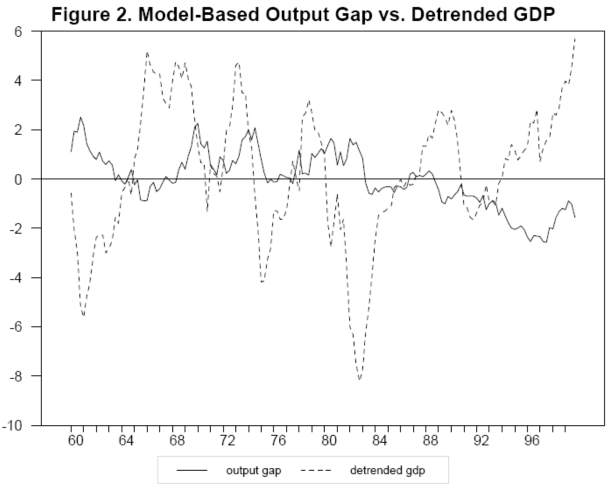


Figure: Galí NBER 2002, fig 2.

Comparison of models

- Idea: Go back one step to

$$\pi_t = \beta E_t \left\{ \pi_{t+1} \right\} + \lambda \widehat{mc}_t$$

- Galí and Gertler JME 1999
  - Show marginal costs proportional to labour's share of income, s_t. Hence, $\widehat{mc}_t = \widehat{s}_t$
  - Conclusion: Backward-looking behaviour quantatitively unimportant
- Sbordone JME 2002
  - Measure of real marginal costs based on wage and productivity statistics
  - Forecast marginal costs and determine path of prices consistent with firms' optimal pricing problem
  - Compare to actual price path of aggregate price level of same period
  - Finds close fit between predictions of the Calvo pricing model and actual inflation

Comparison of models

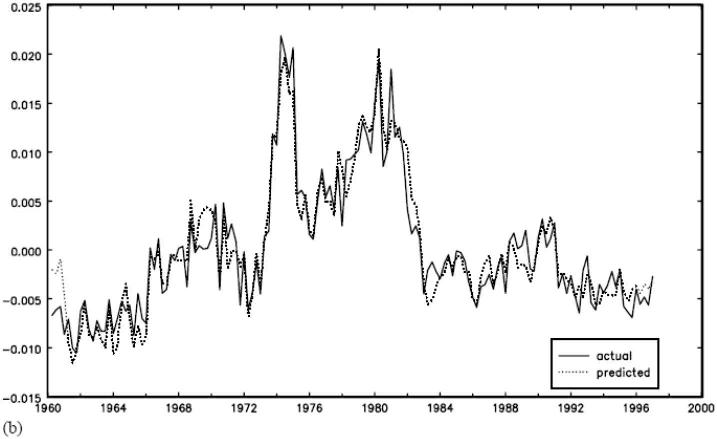


Figure: Sbordone JME 2002, fig. 2 (b), Inflation actual vs. sticky price model.

Lecture 3:

The Design of Monetary Policy

The goals of monetary policy

- What should be the goal of monetary policy?
- Often two goals:
    - Low and stable inflation
    - Stable economic activity
- Often welfare loss function of the form (e.g. Barro and Gordon JME 1983)

$$E_0 \sum_{t=0}^{\infty} \beta^t L_t$$

where

$$L_t = k_y \left(y_t - y^*\right)^2 + k_\pi \left(\pi_t - \pi^*\right)^2$$

The goals of monetary policy

- Loss function:

$$L_t = L_t = k_y \left(y_t - y^*\right)^2 + k_\pi \left(\pi_t - \pi^*\right)^2$$

- Problems:
  - Ad hoc inclusion of inflation and output
  - Choice of target values π^* and y^*
  - Choice of weights k_y and k_π
  - Stabilise inflation or price level?
  - Stabilise output relative to trend or some time-varying potential?
  - Expected and unexpected deviations equally costly?
  - Stabilise over which horizon horizon?

The goals of monetary policy

- Alternative persued here: Monetary policy based on utility-theoretic welfare measure
- Assumes benevolent monetary policy maker seeking to maximise the utility function of the representative household
- ...or alternatively off-set distortions (sources of suboptimality) in the economy
- Can derive a loss function similar to the above as a quadratic approximation to expected utility function
- Implication: Answers to questions given by theory (microfoundation for loss function)

The efficient allocation

- The benevolent social planner solves

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t\right) \quad \text{s.t.} \quad \begin{array}{l} C_t\,(i) = A_t N_t\,(i)^{1-\alpha} \\ N_t = \int_0^1 N_t\,(i)\,di \\ C_t = \left[\int_0^1 C_t\,(i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \end{array}$$

- Optimality/efficiency conditions

$$\begin{aligned} C_t\,(i) &= C_t \\ N_t\,(i) &= N_t \\ -\frac{U_{n,t}}{U_{c,t}} &= MPN_t \\ U_{m,t} &= 0 \end{aligned}$$

Distortion 1: Monopolistic competition

- Prices set as mark-up over marginal costs

$$P_t = \mathcal{M} \frac{W_t}{MPN_t}$$

where

$$\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1} > 1$$

- Labour market equilibrium

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

- Implications
  - Third efficiency condition violated
  - Output and employment inefficiently low (labour demand "shifted down" by $\mathcal{M}$)

Distortion 1: Monopolistic competition

- An ad valorem employment subsidy $\tau = \varepsilon^{-1}$ may undo effects of monopolistic substitution

$$P_t = \mathcal{M} \frac{W_t \left(1 - \tau\right)}{MPN_t}$$

- Labour market equilibrium

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}\left(1 - \tau\right)} = MPN_t$$

- Will generally assume that such an employment subsidy is in place (to isolate effects from other distortions)
- Note: Sales subsidy of $\tau = \left(\varepsilon - 1\right)^{-1}$ would do the same trick

Distortion 2: Average mark-up (sticky prices)

- The average mark-up at any given time determined by the prices set by firms

$$\mathcal{M}_t = P_t \frac{MPN_t}{(1-\tau)\, W_t} = P_t \mathcal{M} \frac{MPN_t}{W_t}$$

- Labour market equibrium

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{\mathcal{M}}{\mathcal{M}_t} MPN_t$$

- Third efficiency condition violated unless average mark-up stabilised at the desired level:

$$\mathcal{M}_t = \mathcal{M}$$

Distortion 3: Relative prices (staggered price-setting)

- Calvo price setting implies that for some pairs of goods (i, j)

$$\begin{aligned} & P_t(i) \neq P_t(j) \\ \Rightarrow \quad & C_t(i) \neq C_t(j) \\ \Rightarrow \quad & N_t(i) \neq N_t(j) \end{aligned}$$

- First and second efficiency condition violated
- ...unless mark-ups are identical across firms (in addition to being equal to the desired level)

Distortion 4: Transaction friction

- Money in utility function gives households a demand for money even when they are dominated in return by bonds
- Motivation: Transaction services
- First-order condition for money in the representative household's problem:

$$U_{m,t} = U_{c,t} \left[1 - \exp\left(-i_t\right)\right]$$

- Forth efficiency condition violated unless monetary policy follows the Friedman rule: $i_t = 0$
  - Opportunity cost of holding money equal to the social cost of producing money
- Will assume that transaction friction negligible

Distortions in sum

- Monopolistic competition distortion: Offset by employment subsidy
- Transaction friction: Negligible
- Two distortions from nominal rigidities
  - Average mark-up distortion: Inability to adjust prices each period implies deviation of mark-ups from optimal level
  - Relative price distortion: Staggered price-setting implies co-existence of different prices and hence output levels for goods entering preferences symmetrically (inefficient allocation of resources)
- With employment subsidy, the flexible-price equilibrium is efficient!
- It turns out that the two distortions from nominal rigidities can be fully off-set by a clever central bank!

Optimal monetary policy

- Assume no initial price distortions (Yun AER 2005 analysis general case) and efficient initial allocation due to employment subsidy
- The efficient allocation can be attained if marginal costs are stabilised at the level consistent with firms' desired mark-up given existing prices
- This implies that the constraints represented by nominal rigidities become non-binding as no firm has an incentive to change its price:

$$\begin{aligned} & P_t^* = P_{t-1} \\ \Rightarrow \quad & P_t = P_{t-1} \end{aligned}$$

- The price level is stabilised (no price distortions emerge)
- Equilibrium output and employment stay at efficient level

Optimal monetary policy

- New Keynesian Phillips curve (AS):

$$\pi_t = \beta E_t \left\{ \pi_{t+1} \right\} + \kappa \tilde{y}_t$$

- Dynamic IS equation:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma^{-1} \left[i_t - E_t \left(\pi_{t+1} \right) - r_t^n \right]$$

- Equilibrium under optimal policy

$$\begin{aligned} \pi_t &= 0 \\ \tilde{y}_t &= 0 \\ i_t &= r_t^n \end{aligned}$$

- Price stability emerges as a feature of the optimal policy even though no weight atteached to such an objective a priori
- Stabilising output is not in itself desirable (should vary one for one with the natural level of output)

Optimal monetary policy

- With $i_t = r_t^n$ the system in the form of Blanchard and Kahn EC 1980 becomes

$$\begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + \sigma^{-1}\beta^{-1}\kappa & \sigma^{-1}\beta^{-1} \\ -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix}$$

- Clearly $(\tilde{y}_t, \pi_t) = (0, 0)$ is a solution to this system
- Problem: Can show that condition for determinacy not satisfied!
- Implication: Continuum of solutions around $(0, 0)$
- Simply tracking the natural interest rate does not guarentee that the optimal allocation is attained (though it is consistent with it)

Optimal monetary policy

- Solve the problem by letting the central bank follow the rule

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \tilde{y}_t$$

- Hence

$$\begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + \sigma^{-1} \left(\phi_y + \frac{\kappa}{\beta} \right) & \sigma^{-1} \left(\phi_\pi - \beta^{-1} \right) \\ -\beta^{-1} \kappa & \beta^{-1} \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix}$$

- Again, $(\tilde{y}_t, \pi_t) = (0, 0)$ is a solution to this system
- Bullard and Mitra JME 2002 show that Blanchard-Kahn conditions satisfied provided a generalised *Taylor principle* is satisfied

$$\kappa \left(\phi_\pi - 1 \right) + (1 - \beta) \phi_y > 0$$

- Implication: $i_t = r_t^n$

Optimal monetary policy

- Alternatively, the central bank may follow a forward-looking rule

$$i_t = r_t^n + \phi_\pi E_t \pi_{t+1} + \phi_y E_t \tilde{y}_{t+1}$$

- Bullard and Mitra JME 2002 show that $(\tilde{y}_t, \pi_t) = (0, 0)$ is the unique solution to this system under the conditions

$$\begin{aligned} \kappa(\phi_\pi - 1) + (1 - \beta)\phi_y &> 0 \\ \kappa(\phi_\pi - 1) + (1 - \beta)\phi_y &< 2\sigma(1 + \beta) \end{aligned}$$

- The determinacy region is smaller in this case (now also an upper bound on coefficients in policy rule)

Optimal monetary policy

- The previous monetary policy rules are examples of *optimal rules*
- Shortcoming: A central bank must be able to observe the natural rate of interest (though not necessarily the natural rate of output). This requires knowledge of
  - the true model
  - true parameter values
  - realised shocks
- Other caveats
  - High interest rate volatility may exacerbate transaction frictions (Woodford, 2003, ch. 6)
  - May not be feasible if natural rate of interest becomes negative due to zero lower bound on nominal interest rates (Eggertsson and Woodford BPEA 2003)
  - Flexible-price equilibrium assumed efficient: No trade-off between inflation and output stabilisation

Simple rules and the welfare loss function

- Alternative to optimal rules are *simple rules* that depend on observable variables only
  - Do not require knowledge of true parameter values
  - Ideally approximate optimal policy rules across different models
  - ...but generally suboptimal (do not track natural rate)
- Alternative simple rules may be evaluated using a utility-theoretic welfare criterion in the form of a quadratic approximation to expected utility
  - Mathematically convenient: goes nicely with the linearised structural equations
  - Need higher-order approximations of structural equations to evaluate higher-order approximations of welfare
  - Comparability with the traditional literature on monetary policy evaluation
- Caveats concerned with accuracy (Woodford, 2003, ch. 6)

Welfare loss function

- Take second-order Taylor approximation of utility function $U\left(C_t, \frac{M_t}{P_t}, N_t\right)$ around the efficient steady state
- Use equilbrium conditions to replace terms in consumption with terms in output
- Use production function and Dixit-Stiglitz demand functions to replace terms in hours with terms in price dispersion, technology shocks and output
- Disregard terms independent of policy and do a bit of trivial but tedious algebra to get an expression for the utility loss expressed as a fraction of steady-state consumption $\left(U_t - U\right) / U_c C$
- The welfare function can now be written as

$$E_0 \sum_{t=0}^{\infty} \beta \frac{U_t - U}{U_c C}$$

- Establish relation between price dispersion and inflation

Welfare loss function

- The resulting welfare *loss* function is

$$-E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_c C} = E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{\varepsilon}{\lambda} \pi_t^2 \right]$$

- From this it follows that the *expected average period welfare loss* expressed as a fraction of steady-state consumption is

$$\mathcal{W} = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) VAR_0 \left(\tilde{y}_t \right) + \frac{\varepsilon}{\lambda} VAR_0 \left(\pi_t \right) \right]$$

- Note: The optimal allocation leads to a zero loss

Welfare loss function

- The standard ad hoc function:

$$L_t = k_y \left(y_t - y^*\right)^2 + k_\pi \left(\pi_t - \pi^*\right)^2$$

- The loss function just derived from first principles:

$$2L_t = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \tilde{y}_t^2 + \frac{\varepsilon}{\lambda} \pi_t^2$$

- Theoretical justification for loss function with specific choices
  - Attention to variation in inflation rather than price level
  - Inflation variation costly, not just unexpected part
  - Optimal inflation target: $\pi^* = 0$
  - Output stabilisation around $y^* = y_t^n$
  - Weights depend on model parameters, e.g.

$$\frac{\partial k_\pi}{\partial \theta} = \frac{\partial k_\pi}{\partial \lambda} \frac{\partial \lambda}{\partial \theta} > 0$$

Simple monetary policy rules

- Taylor CRSPP 1993 suggests that the following simple rule is a good approximation to US monetary policy

$$i_t = 4 + 1.5\left(\pi_t - 2\right) + 0.125\hat{y}_t$$

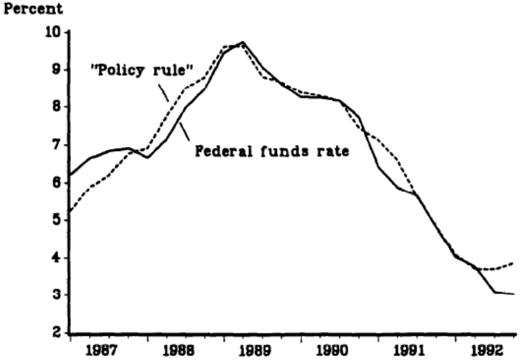


Figure 1. Federal funds rate and example policy rule.

Simple monetary policy rules

- Taylor rule

$$\begin{aligned} i_t &= \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t \\ &= \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_y \hat{y}_t^n \end{aligned}$$

  where $\phi_\pi, \phi_y > 0$ are such that the determinacy conditions are satisfied

- Equilibrium dynamics

$$\begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} + \mathbf{B} \left(r_t^n - \phi_y \hat{y}_t^n \right)$$

  where the only driving force is the technology shock

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

- With $\phi_y > 0$ the central bank can only replicate the flexible price equilibrium in the special case $\phi_y = \sigma \left(1 - \rho_a\right) \Rightarrow r_t^n - \phi_y \hat{y}_t^n$

Simple monetary policy rules

	Taylor				-	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_y	0.125	0	0	1	-	-
-	-	-	-	-	-	-
$\sigma\left(\tilde{y}\right)$	0.55	0.28	0.04	1.40	-	-
$\sigma\left(\pi\right)$	2.60	1.33	0.21	6.55	-	-
L	0.30	0.08	0.002	1.92	-	-

Simple monetary policy rules

- Friedman (1960) suggest a simple money growth rule (the 2nd Friedman rule)

$$\Delta m_t = 0$$

- We therefore need the money demand relation to characterise equilibrium dynamics

$$m_t - p_t = c_t - \eta i_t + \zeta_t$$

where we have assumed that $\nu = \sigma = 1$ and where ζ_t is a money demand shock

$$\Delta \zeta_t = \rho_\zeta \Delta \zeta_{t-1} + \varepsilon_t^\zeta$$

- Equilibrium dynamics where $l_t^+ = m_t - p_t - \zeta_t$

$$\begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \pi_{t+1} \\ l_t^+ \end{bmatrix} = \mathbf{A}_m \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ l_{t-1}^+ \end{bmatrix} + \mathbf{B}_m \begin{bmatrix} r_t^n \\ \hat{y}_t^n \\ \Delta \zeta_t \end{bmatrix}$$

Simple monetary policy rules

	Taylor				Friedman	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_π	0.125	0	0	1	-	-
$(\sigma_\zeta, \rho_\zeta)$	-	-	-	-	$(0, 0)$	$(0.0063, 0.6)$
$\sigma(\tilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
L	0.30	0.08	0.002	1.92	0.08	0.38

Simple monetary policy rules

- Clarida, Galí and Gertler QJE 2000

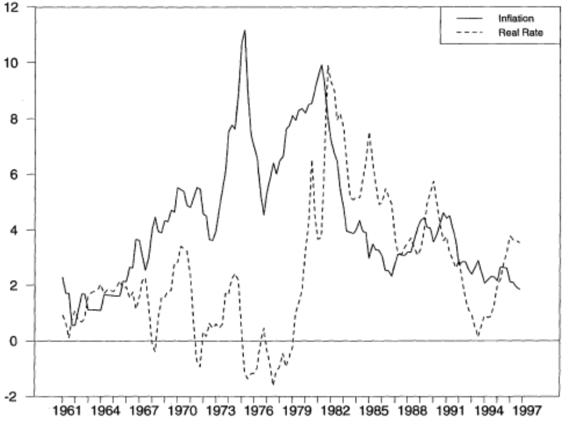


FIGURE IV
The Real Interest Rate and Inflation

Simple monetary policy rules

- Clarida, Galí and Gertler QJE 2000 estimate

$$i_t = \rho i_{t-1} + (1-\rho)\left[i + \beta E_t\left(\pi_{t+1} - \pi^*\right) + \gamma E_t\left(y_{t+1} - y^*_{t+1}\right)\right]$$

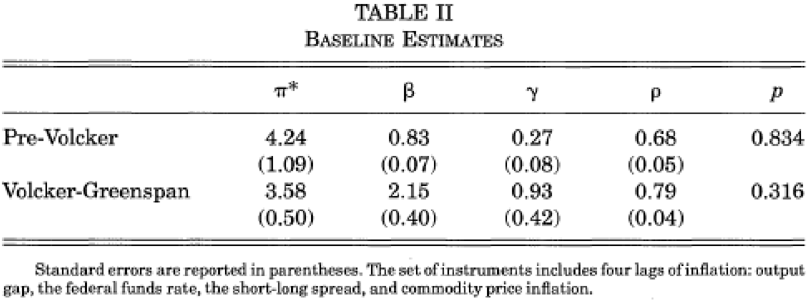


TABLE II

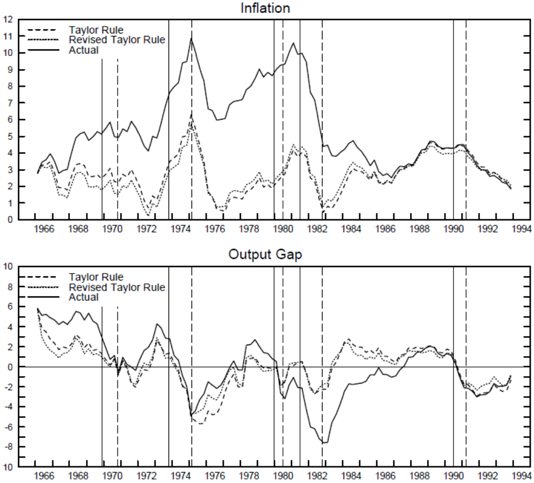
BASELINE ESTIMATES

	π^*	β	γ	ρ	p
Pre-Volcker	4.24	0.83	0.27	0.68	0.834
	(1.09)	(0.07)	(0.08)	(0.05)	
Volcker-Greenspan	3.58	2.15	0.93	0.79	0.316
	(0.50)	(0.40)	(0.42)	(0.04)	

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.

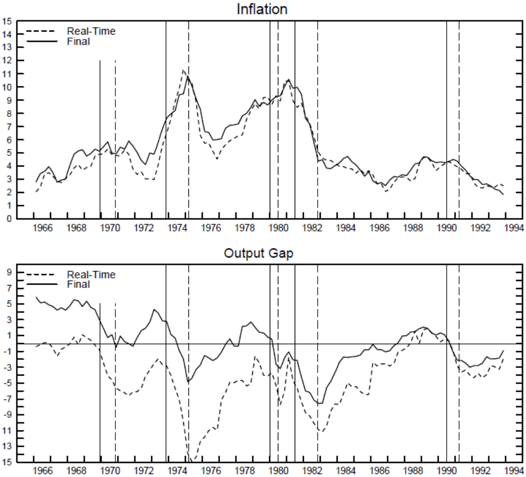
Simple monetary policy rules

- Orphanides JME 2003 runs counterfactual with standard Taylor rule...



Simple monetary policy rules

- ...but identifies real-time misperception of output gap...



Simple monetary policy rules

- ...leading monetary policy to be too easy despite following the Taylor rule pre-Volcker (and stighter than the rule prescribes during the following Volcker disinflation)

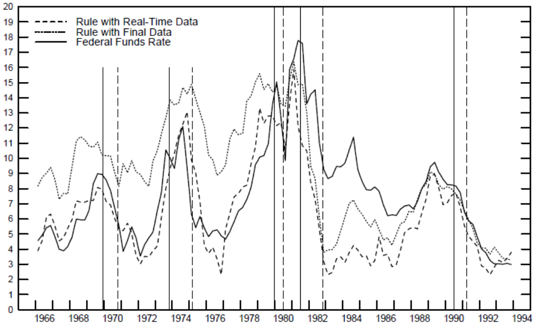


Fig. 5. Then and now: Taylor rule with final and real-time data.

Discretion versus commitment

- So far the flexible price equilibrium has been efficient
  - Employment subsidy (otherwise inefficiently low natural output leads to *inflation bias*)
  - Negligible transaction frictions (otherwise welfare costs from interest rate volatility)
- Now suppose there are real imperfections that make the flexible price equilibrium (but not the steady state inefficient)
  - Variations in desired price mark-ups
  - Variations in wage mark-ups
  - Fluctuations in labour income taxes
- If y_t^e is the efficient output level, then the welfare relevant output gap will be

$$x_t \equiv y_t - y_t^e$$

- Implication:

$$\tilde{y}_t = x_t + (y_t^e - y_t^n)$$

Discretion versus commitment

- We may use this to rewrite the model as

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t \\ x_t &= E_t x_{t+1} - \sigma^{-1}\left[i_t - E_t \pi_{t+1} - r_t^e\right]\end{aligned}$$

  where $u_t \equiv \kappa\left(y_t^e - y_t^n\right)$ is referred to as a *cost-push shock* and

$$r_t^e = \rho + \sigma E_t\left(\Delta y_{t+1}^e\right)$$

  is the *efficient interest rate* (invariant to monetary policy)
- The cost-push shock increases inflation for given output gap (exogenous w.r.t. monetary policy)
- Generates a trade-off for the central bank!
- Assumption:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

Discretion versus commitment

- The welfare loss function becomes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \frac{\kappa}{\varepsilon} x_t^2 \right]$$

- The monetary policy problem is to choose paths for π_t and x_t to minimise this function subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

- Under discretion, the central bank chooses π_t and x_t period-by-period
- Under commitment, the central bank is able to credible commit to a state-contingent path $\{\pi_t, x_t\}_{t=0}^{\infty}$

Discretion versus commitment

- Under discretion, the central bank solves

$$\begin{aligned} &\min \pi_t^2 + \frac{\kappa}{\varepsilon} x_t^2 \\ &\text{s.t. } \pi_t = \kappa x_t + v_t \end{aligned}$$

  where $v_t = \beta E_t \pi_{t+1} + u_t$ is taken as given
- Optimality condition gives a *targeting rule*

$$x_t = -\varepsilon \pi_t$$

- Equilibrium

$$\begin{aligned} \pi_t &= \frac{1}{\varepsilon\kappa + (1 - \beta\rho_u)} u_t \\ x_t &= -\frac{\varepsilon}{\varepsilon\kappa + (1 - \beta\rho_u)} u_t \end{aligned}$$

- Trade-off: Deviations from targets in proportion to cost-push shock

Discretion versus commitment

- From the dynamic IS equation, the equilibrium interest rate becomes

$$i_t = r_t^e + \frac{\varepsilon\sigma\left(1-\rho_u\right)+\rho_u}{\varepsilon\kappa+\left(1-\beta\rho_u\right)}u_t$$

- Usual problem: Does not guarantee unique solution
- Alternative rule ensuring determinacy with $\phi_\pi > 1$:

$$i_t = r_t^e + \phi_\pi \pi_t$$

- But efficient interest rate unobservable
- Some authors therefore emphasise *targeting rules* rather than *instrument rules*

Discretion versus commitment

- Under commitment the central bank solves

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \frac{\kappa}{\varepsilon} x_t^2 \right]$$
$$\text{s.t. } \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

- Optimality condition (by Langrangian method)

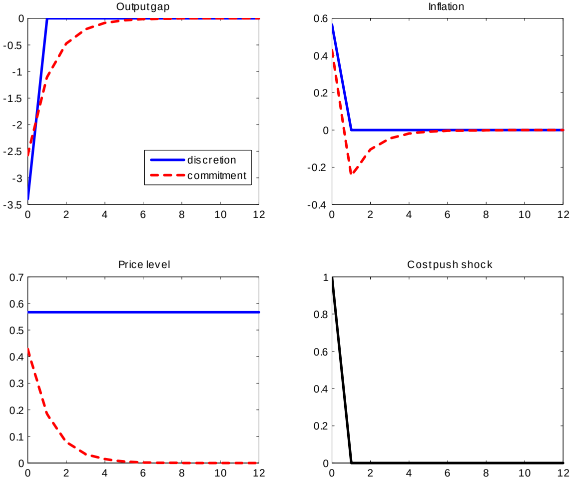
$$x_t = \varepsilon \hat{p}_t$$

- Note: Price-level targeting!
- We skip the solution as it requires a bit of work...

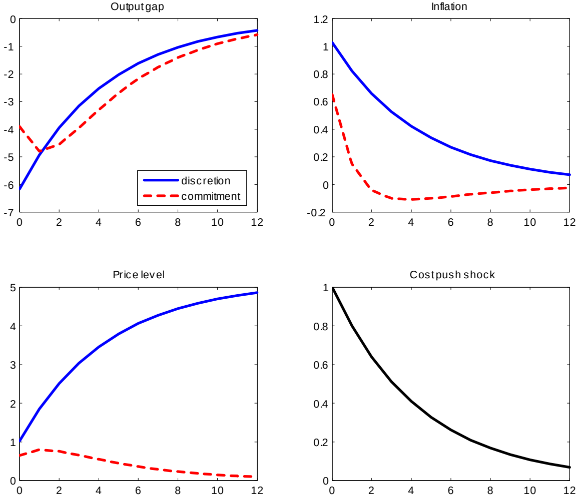
Discretion versus commitment

- The central bank can offset the inflation impact of a cost-push shock by
    - lowering current output gap
    - committing to lowering future output gaps
- Discretionary policy leads to a *stabilisation bias*
    - Future output gaps stabilised too much
    - Current output gap and inflation moves too much
- Commitment enables central bank to imporve the output gap/inflation tradeoff when the shock hits

Discretion versus commitment



Discretion versus commitment



Lecture 3:

The Small Open Economy

The open economy

- Extending to the open economy adds new effects
    - Trade and capital flows
    - Wedge between output and consumption
    - Exchange rate movements
    - Wedge between domestic and consumer prices
    - Shocks and spillovers from other countries (real and policy)
- New modelling choices
    - Relative size of economies
    - Degree of substitution of goods produced in different countries
    - Nature of international asset markets
    - Firms' ability to price discriminate across countries
    - Invoicing currency
    - Policy coordination

The open economy

- *New* open macroeconomics initiated by Obstfeld and Rogoff JPE 1995 (stochastic version: JIE 2000)
  - Two-country model
  - DSGE with monopolistic competition
  - Prices set one period in advance
- Extention of New Keynesian model to open economies
  - Calvo-pricing
  - Model summarised by Phillips curve and expectational IS relation
  - Emphasis on monetary policy through interest rate rule
- Approaches merging
  - Clarida, Galí and Gertler JME 2002: Two-country version of basic New Keynesian model from lecture one

The open economy

- Here consider *small open economy* version of Galí and Monacelli RES 2005
- Small open economy defined as economy too small to affect world prices and activity
    - No room for strategic interaction by policy makers
- The small open economy is one of a continuum of infinitesimally small economies making up the world economy
    - Alternative: Small open economy modelled by treating foreign variables as exogenous
- Results in New Keynesian Phillips curve and dynamic IS relation where coefficients depend on parameters specific to the open economy

The open economy

- Derive loss function as quadratic approximation to household utility *for special case*
- In loss function enter *domestic* (producer) inflation and output gap
- Optimal policy requires that *domestic* price level is fully stabilised
- Consider the simple rules
    - Taylor rule with *domestic* inflation (ranked one)
    - Taylor rule with *CPI* inflation (ranked two)
    - Exchange rate peg (ranked third)
- Kirsonova, Leith and Wren-Lewis EC 2006: The special case loss function also includes the exchange rate if shocks to uncovered interest rate parity
- De Paoli JIE 2009: Loss function includes exchange rate movements in the *general case*

A small open economy

- World economy consists of continuum of countries $i \in [0,1]$ with measure zero
- Domestic decisions have no impact on the rest of word
- Countries are identical
- Exceptions are imperfectly correlated productivity shocks
- Each country produces a continuum of good indexed by $j \in [0,1]$
- Households have access to a complete set of contingent claims traded internationally
- The economy is cashless (i.e. we abstract from money)

Households

- Representative households decision problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

where real consumption is

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t} \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

  - $\alpha \in [0,1]$ is inversely related to home bias in preferences, i.e., measure of *openness*
  - $\eta > 0$ measures substitutability between domestic and foreign goods

Households

- Consumption indices

$$
\begin{aligned}
C_{H,t} &= \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \\
C_{i,t} &= \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \\
C_{F,t} &= \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}}\, di\right)^{\frac{\gamma}{\gamma-1}}
\end{aligned}
$$

  - $\varepsilon > 1$ is elasticity of substitution between products produced within one country
  - $\gamma > 0$ is elasticity of substitution between between imported goods produced in different countries

Households

- $P_{i,t}(j)$ is the price of good j produced in country i denoted in domestic currency
- Intratemporal decision problem gives rise to demand functions

$$\begin{aligned} C_{H,t}(j) &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \\ C_{i,t}(j) &= \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t} \end{aligned}$$

where the *domestic* price index and the price index of goods imported from country i, respectively, are

$$\begin{aligned} P_{H,t} &= \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}} \\ P_{i,t} &= \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}} \end{aligned}$$

Households

- In addition

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t}$$

where

$$P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$$

is the *imported goods* price index

Households

- Finally

$$\begin{aligned} C_{H,t} &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \\ C_{F,t} &= \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \end{aligned}$$

where the *consumer price index (CPI)* is

$$P_t = \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

  - If $P_{H,t} = P_{F,t}$ then α is the share of domestic consumption allocated to imported goods (cf. degree of openness)

Households

- Hence, total consumption spending becomes

$$
\begin{aligned}
& \int_0^1 P_{H,t}(j)\, C_{H,t}(j)\, dj + \int_0^1 \int_0^1 P_{i,t}(j)\, C_{i,t}(j)\, dj di \\
= \;& P_{H,t} C_{H,t} + \int_0^1 P_{i,t} C_{i,t} di \\
= \;& P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \\
= \;& P_t C_t
\end{aligned}
$$

Households

- Complete financial markets assumed: A full set of contingent claims available to households
- Let $D_{t+1}\left(\varsigma^{t+1}\right)$ is nominal payoff of claims (state-contingent!)
- Let $V_{t,t+1}\left(\varsigma^{t+1}\right)$ be the period-t price (in domestic currency) of a claim to one unit of domestic currency in state ς^{t+1} (and in state ς^{t+1} only).
- Define the stochastic discount factor

$$Q_{t,t+1}\left(\varsigma^{t+1}\right)=\frac{V_{t,t+1}\left(\varsigma^{t+1}\right)}{\xi_{t,t+1}\left(\varsigma^{t+1}\right)}$$

- An allocation of resources to a portfolio of assets can now be found as (suppressing dependence on states)

$$\sum_{\varsigma^{t+1}} V_{t,t+1}D_{t+1}=\sum_{\varsigma^{t+1}}\xi_{t,t+1}\frac{V_{t,t+1}}{\xi_{t,t+1}}D_{t+1}=E_t\left(Q_{t,t+1}D_{t+1}\right)$$

Households

- If $D_{t+1}\left(\varsigma^{t+1}\right) = 1$ for all ς^{t+1} then

$$\sum_{\varsigma^{t+1}} V_{t,t+1} D_{t+1} = \sum_{\varsigma^{t+1}} \xi_{t,t+1} \frac{V_{t,t+1}}{\xi_{t,t+1}} = E_t\left(Q_{t,t+1}\right) \equiv Q_t$$

- Consequently, the gross risk-free nominal interest rate is defined as

$$1 + \mathcal{I}_t = \frac{1}{Q_t}$$

Households

- Households optimise life-time utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

subject to a sequence of flow budget constraints

$$P_t C_t + E_t \left(Q_{t,t+1} D_{t+1} \right) \leq D_t + W_t N_t + T_t$$

Households

- Consumption f.o.c.

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\beta C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}}$$

- Financial assets f.o.c.:

$$\frac{\lambda_{t+1}}{\lambda_t} = Q_{t,t+1}$$

- Combining (and taking probability-weighted sum) gives Euler equation

$$Q_t = E_t \left(\frac{\beta C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right)$$

Households

- Labour f.o.c. (combined with consumption f.o.c.):

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{C_t^{-\sigma}}$$

Households

- Linearised first-order conditions:

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t \\ c_t &= E_t c_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \rho\right) \end{aligned}$$

where $\pi_t = p_t - p_{t-1}$ is *CPI* inflation

  - In open economy, wedge between CPI and domestic price index
  - CPI essential for households' decision problems
  - Relation?

Firms

- Each country has a continuum of firms $j \in [0, 1]$ producing a differentiated good according to

$$Y_t\left(j\right) = A_t N_t\left(j\right)$$

where $a_t = \rho_a a_{t-1} + \varepsilon_t$

- Marginal costs are common across firms due to linear production technology (and common factor market)

$$NMC_t = \frac{\left(1-\tau\right) W_t}{MPN_t} = \frac{\left(1-\tau\right) W_t}{A_t}$$

- In logs

$$\begin{aligned} y_t &= a_t + n_t \\ mc_t &= -v + w_t - p_{H,t} - a_t \end{aligned}$$

where $v \equiv \ln\left(1-\tau\right)$

Firms

- Calvo price-setting: Each period a firm resets its price with fixed probability $(1-\theta)$
- $\bar{p}_{H,t}$ is the chosen price
- Law of motion of domestic price level

$$p_{H,t} = \theta p_{H,t-1} + (1-\theta)\,\bar{p}_{H,t}$$

- Log-linear approximation of f.o.c.:

$$\bar{p}_{H,t} = \mu + (1-\beta\theta)\sum_{k=0} (\beta\theta)^k E_t\left(mc_{t+k} + p_{H,t+k}\right)$$

- Combine to get

$$\pi_{H,t} = \beta E_t\left(\pi_{H,t+1}\right) + \lambda \widehat{mc}_t$$

where $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$

International bookkeeping

- Log-linearising *CPI* around a symmetric steady state yields

$$p_t = (1-\alpha)\, p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t$$

where $s_t \equiv p_{F,t} - p_{H,t}$ is the (log) effective *terms of trade*

- *Domestic* inflation:

$$\pi_{H,t} = p_{H,t} - p_{H,t-1}$$

- Combine to get relation to *CPI* inflation

$$\begin{aligned}\pi_t &= p_t - p_{t-1} \\ &= p_{H,t} + \alpha s_t - \left(p_{H,t-1} + \alpha s_{t-1}\right) \\ &= \pi_{H,t} + \alpha \Delta s_t\end{aligned}$$

  - Gap between the two measures of inflation proportional to change in terms of trade
  - α is coefficient of proportionality

International bookkeeping

- The law of one price holds by assumption

$$P_{i,t}\left(j\right)=\mathcal{E}_{i,t}P_{i,t}^{i}\left(j\right)$$

- Implications

$$P_{i,t}=\left(\int_{0}^{1}\left(\mathcal{E}_{i,t}P_{i,t}^{i}\left(j\right)\right)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}=\mathcal{E}_{i,t}P_{i,t}^{i}$$

$$\Rightarrow\quad P_{F,t}=\left(\int_{0}^{1}\left(\mathcal{E}_{i,t}P_{i,t}^{i}\right)^{1-\gamma}di\right)^{\frac{1}{1-\gamma}}$$

- Log-linearising gives

$$p_{F,t}=e_{t}+p_{t}^{*}$$

where $e_t=\int_o^1 e_{it}di$ is the *effective nominal exchange rate* and $p_t^*=\int_o^1 p_{i,t}^i di$ the *world price index rate*

International bookkeeping

- Define the *bilateral real exchange rate* as

$$\mathcal{Q}_{i,t} = \frac{\mathcal{E}_{i,t} P_t^i}{P_t}$$

- Define the (log) *effective real exchange rate* as

$$q_t = \int_0^1 q_{i,t} di = \int_0^1 \left(e_{i,t} + p_t^i - p_t \right) di = e_t + p_t^* - p_t$$

- Combine previous expressions to get relation between real exchange rate and terms of trade

$$\left. \begin{aligned} q_t &= e_t + p_t^* - p_t \\ p_{F,t} &= e_t + p_t^* \\ s_t &= p_{F,t} + p_{H,t} \\ p_t &= p_{H,t} + \alpha s_t \end{aligned} \right\} \Rightarrow \quad \begin{aligned} q_t &= e_t + p_t^* - p_t \\ &= p_{F,t} - p_t \\ &= s_t - p_{H,t} - p_t \\ &= (1-\alpha) s_t \end{aligned}$$

International bookkeeping

- $\mathcal{E}_{i,t}$ is the bilateral nominal exchange rate: Price of country i's currency in terms of domestic currency
- Let domestic currency be krona and country i's currency be dollar
- Consider two trading strategies to get one dollar tomorrow:
  - Convert to dollar and buy dollar asset today: $V^i_{t,t+1}$ is dollar price and $\mathcal{E}_{i,t} V^i_{t,t+1}$ is the krona price of one dollar tomorrow in the foreign market
  - Buy krona asset today and convert to dollar tomorrow: Need $\mathcal{E}_{i,t+1}$ krona to get one dollar tomorrow so must buy $\mathcal{E}_{i,t+1}$ claims paying one krona tomorrow at total krona price $V_{t,t+1}\mathcal{E}_{i,t+1}$
- Since there's only one international financial market, perfect arbitrage implies

$$\mathcal{E}_{i,t} V^i_{t,t+1} = V_{t,t+1}\mathcal{E}_{i,t+1}$$

International bookkeeping

- The no-arbitrage condition

$$\begin{aligned} \mathcal{E}_{i,t} V^i_{t,t+1} &= V_{t,t+1}\mathcal{E}_{i,t+1} \\ \Leftrightarrow \quad \mathcal{E}_{i,t} Q^i_{t,t+1} &= Q_{t,t+1}\mathcal{E}_{i,t+1} \end{aligned}$$

implies an uncovered interest rate parity (UIP) condition

$$E_t\left(Q^i_{t,t+1}\frac{\mathcal{E}_{i,t}}{\mathcal{E}_{i,t+1}}\right) = E_t\left(Q_{t,t+1}\right)$$

- In logs

$$i_t = i^i_t + E_t\Delta e_{it+1}$$

or

$$i_t = i^*_t + E_t\Delta e_{t+1}$$

where $i^*_t = \int_o^1 i^i_t di$ the *world interest rate*

International bookkeeping

- Combine asset and consumption f.o.c

$$Q_{t,t+1} = \frac{\lambda_{t+1}}{\lambda_t} = \frac{\beta C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}}$$

(for domestic and foreign country) with no-arbitrage condition

$$\mathcal{E}_{i,t} Q^i_{t,t+1} = Q_{t,t+1} \mathcal{E}_{i,t+1}$$

to get

$$\begin{aligned} \frac{\beta \left(C^i_{t+1}\right)^{-\sigma}}{\left(C^i_t\right)^{-\sigma}} \frac{P^i_t}{P^i_{t+1}} \mathcal{E}_{i,t} &= \frac{\beta C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \mathcal{E}_{i,t+1} \\ \Leftrightarrow \frac{C_{t+1}}{C_t} \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} &= \frac{C^i_{t+1}}{C^i_t} \mathcal{Q}_{i,t+1}^{\frac{1}{\sigma}} \end{aligned}$$

International bookkeeping

- Assuming identical initial conditions so that $C_0^i = C_0$ and $\mathcal{Q}_0 = 1$, repeated substitution in

$$\frac{C_{t+1}}{C_t} \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} = \frac{C_{t+1}^i}{C_t^i} \mathcal{Q}_{i,t+1}^{\frac{1}{\sigma}}$$

gives

$$C_1 = C_1^i \mathcal{Q}_{i,1}^{\frac{1}{\sigma}} \Rightarrow C_2 = C_2^i \mathcal{Q}_{i,2}^{\frac{1}{\sigma}} \Rightarrow \ldots \Rightarrow C_t = C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}}$$

- Taking logs and integrating over i gives

$$\begin{aligned} c_t &= c_t^* + \frac{1}{\sigma} q_t \\ &= c_t^* + \frac{1-\alpha}{\sigma} s_t \end{aligned}$$

International bookkeeping: Summary

- Terms of trade

$$s_t = p_{F,t} - p_{H,t}$$

- CPI inflation

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t$$

- Law of one price

$$p_{F,t} = e_t + p_t^*$$

- Effective real exchange rate

$$q_t = e_t + p_t^* - p_t = (1 - \alpha) s_t$$

- Uncovered interest rate parity

$$i_t = i_t^* + E_t \Delta e_{t+1}$$

- Consumption risk sharing

$$c_t = c_t^* + \frac{1}{\sigma} q_t = c_t^* + \frac{1-\alpha}{\sigma} s_t$$

Market clearing

- Market clearing condition for domestic goods

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j)\, di$$

- Plug this into the definition of aggregate domestic output

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- After substituting in demand functions, take logs to get

$$y_t = c_t + \frac{\alpha\omega}{\sigma} s_t$$

  where $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$

- Aggregating over all countries gives

$$y_t^* = c_t^* \Rightarrow y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t$$

  where $\sigma_\alpha \equiv \sigma \left[1 + \alpha(\omega - 1)\right]^{-1} > 0$

Market clearing

- Net exports in terms of domestic output expressed as a fraction of steady state output is defined as

$$NX_t \equiv \frac{1}{Y}\left(Y_t - \frac{P_t}{P_{H,t}} C_t\right)$$

- In logs

$$nx_t = y_t - c_t - \alpha s_t$$

- Combine with market clearing

$$y_t = c_t + \frac{\alpha\omega}{\sigma} s_t$$

to get

$$nx_t = \alpha\left(\frac{\omega}{\sigma} - 1\right) s_t$$

The open economy model

- Households (labour supply and Euler equation)

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t \\ c_t &= E_t c_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1} - \rho\right) \end{aligned}$$

- Firms (production, price setting and marginal costs)

$$\begin{aligned} y_t &= a_t + n_t \\ \pi_{H,t} &= \beta E_t \left(\pi_{H,t+1}\right) + \lambda \widehat{mc}_t \\ mc_t &= -v + w_t - p_{H,t} - a_t \end{aligned}$$

- Market clearing

$$y_t = c_t + \frac{\alpha\omega}{\sigma} s_t$$

- International bookkeeping relations and monetary policy specification

The open economy model

- Combining Euler equation and market clearing condition

$$\begin{aligned} c_t &= E_t c_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \rho\right) \\ y_t &= c_t + \frac{\alpha\omega}{\sigma} s_t \end{aligned}$$

  to get

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \rho\right) + \frac{\omega\alpha}{\sigma} E_t\left(\Delta s_{t+1}\right)$$

- Using bookkeeping relations, this can be written as

$$y_t = E_t y_{t+1} - \frac{1}{\sigma_\alpha}\left(i_t - E_t \pi_{H,t+1} - \rho\right) + \alpha\Theta E_t\left(\Delta y^*_{t+1}\right)$$

  where $\Theta \equiv \omega - 1$

The open economy model

- Closed economy Euler

$$y_t = -\frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} - \rho\right) + E_t y_{t+1}$$

- Open-economy Euler

$$y_t = E_t y_{t+1} - \frac{1}{\sigma_\alpha}\left(i_t - E_t\pi_{H,t+1} - \rho\right) + \alpha\Theta E_t\left(\Delta y^*_{t+1}\right)$$

- In general

$$\begin{aligned} \Theta &\lesseqgtr 0 \\ sign\left(\frac{\partial\sigma_\alpha}{\partial\alpha}\right) &= sign\left(\Theta\right) \end{aligned}$$

The open economy model

- Combine marginal costs, labour supply and production

$$\begin{aligned} mc_t &= -v + w_t - p_{H,t} - a_t \\ w_t - p_t &= \sigma c_t + \varphi n_t \\ y_t &= a_t + n_t \end{aligned}$$

  to get

$$mc_t = -v + \sigma c_t + \varphi \left(y_t - a_t \right) + p_t - p_{H,t} - a_t$$

- Use bookkeeping relations to get

$$mc_t = -v + \left(\sigma_\alpha + \varphi \right) y_t + \left(\sigma - \sigma_\alpha \right) y_t^* - \left(1 + \varphi \right) a_t$$

The open economy model

- Closed-economy marginal costs (with linear technology)

$$\begin{aligned} mc_t &= -\nu + w_t - p_t - a_t \\ &= -\nu + (\sigma + \varphi)\, y_t - (1 + \varphi)\, a_t \end{aligned}$$

- In the open economy

$$mc_t = -v + (\sigma_\alpha + \varphi)\, y_t + (\sigma - \sigma_\alpha)\, y_t^* - (1 + \varphi)\, a_t$$

- In general

$$\frac{\partial \sigma_\alpha}{\partial \alpha} \lessgtr 0$$

Flexible price solution

- Flexible prices implies $\lambda^{-1} = 0$ so that

$$\pi_{H,t} = \beta E_t \left(\pi_{H,t+1} \right) + \lambda \widehat{mc}_t$$

becomes

$$mc_t + \mu = \mu - v + \left(\sigma_\alpha + \varphi \right) y_t + \left(\sigma - \sigma_\alpha \right) y_t^* - \left(1 + \varphi \right) a_t = 0$$

- Rearranging gives us the natural level of output (assuming $\mu = v$)

$$y_t^n = \frac{v - \mu}{\sigma_\alpha + \varphi} - \frac{\sigma - \sigma_\alpha}{\sigma_\alpha + \varphi} y_t^* + \frac{1 + \varphi}{\sigma_\alpha + \varphi} a_t$$

- Implication

$$\widehat{mc}_t = \left(\sigma_a + \varphi \right) \left(y_t - y_t^n \right) \equiv \left(\sigma_a + \varphi \right) \tilde{y}_t$$

Flexible price solution

- Flexible prices implies $\lambda^{-1} = 0$ so that

$$\pi_{H,t} = \beta E_t \left(\pi_{H,t+1} \right) + \lambda \widehat{mc}_t$$

becomes

$$mc_t + \mu = \mu - v + \left(\sigma_\alpha + \varphi \right) y_t + \left(\sigma - \sigma_\alpha \right) y_t^* - \left(1 + \varphi \right) a_t = 0$$

- Rearranging gives us the natural level of output (assuming $\mu = v$)

$$y_t^n = \frac{v - \mu}{\sigma_\alpha + \varphi} - \frac{\sigma - \sigma_\alpha}{\sigma_\alpha + \varphi} y_t^* + \frac{1 + \varphi}{\sigma_\alpha + \varphi} a_t$$

- Implication

$$\widehat{mc}_t = \left(\sigma_a + \varphi \right) \left(y_t - y_t^n \right) \equiv \left(\sigma_a + \varphi \right) \tilde{y}_t$$

Flexible price solution

- From the Euler equation we get

$$r_t \equiv i_t - E_t \pi_{H,t+1} = \rho + \sigma_\alpha E_t \Delta y_{t+1} + \sigma_\alpha \alpha \Theta E_t \left(\Delta y^*_{t+1} \right)$$

- Using the natural output solution gives the open-economy natural rate of interest

$$\begin{aligned} r^n_t &= \rho + \sigma_\alpha E_t \Delta y^n_{t+1} + \sigma_\alpha \alpha \Theta E_t \left(\Delta y^*_{t+1} \right) \\ &= \rho - \frac{\sigma_\alpha \left(1+\varphi\right)}{\sigma_\alpha + \varphi} \left(1 - \rho_a\right) a_t + \frac{\alpha \Theta \varphi \sigma_\alpha}{\sigma_\alpha + \varphi} E_t \left(\Delta y^*_{t+1} \right) \end{aligned}$$

- In closed economy we have

$$r^n_t = \rho - \frac{\sigma \left(1+\varphi\right)}{\sigma + \varphi} \left(1 - \rho_a\right) a_t$$

Equilibrium dynamics

- Combine price setting and relation between marginal costs and output gap

$$\begin{aligned}\pi_{H,t} &= \beta E_t\left(\pi_{H,t+1}\right) + \lambda \widehat{mc}_t \\ \widehat{mc}_t &= \left(\sigma_a + \varphi\right)\tilde{y}_t\end{aligned}$$

to get open-economy New Keynesian Phillips curve

$$\pi_{H,t} = \beta E_t\left(\pi_{H,t+1}\right) + \kappa_\alpha \tilde{y}_t$$

where

$$\kappa_\alpha = \lambda\left(\sigma_a + \varphi\right) = \frac{\left(1-\beta\theta\right)\left(1-\theta\right)}{\theta}\left(\sigma_a + \varphi\right)$$

Equilibrium dynamics

- Combine Euler equation in general with the one under flexible prices

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \frac{1}{\sigma_\alpha}\left(i_t - E_t \pi_{H,t+1} - \rho\right) + \alpha\Theta E_t\left(\Delta y^*_{t+1}\right) \\
y^n_t &= E_t y^n_{t+1} - \frac{1}{\sigma_\alpha}\left(r^n_t - \rho\right) + \alpha\Theta E_t\left(\Delta y^*_{t+1}\right)
\end{aligned}
$$

to open-economy dynamic IS equation

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma_\alpha}\left(i_t - E_t \pi_{H,t+1} - r^n_t\right)$$

where

$$\sigma_\alpha = \frac{\sigma}{1 + \alpha\left(\omega - 1\right)} = \frac{\sigma}{1 + \alpha\left(\sigma\gamma + \left(1 - \alpha\right)\left(\sigma\eta - 1\right) - 1\right)}$$

Equilibrium dynamics

- Basic New Keynesian Model

$$\begin{aligned}\pi_t &= \beta E_t\left(\pi_{t+1}\right)+\kappa\tilde{y}_t\\ \tilde{y}_t &= E_t\left(\tilde{y}_{t+1}\right)-\frac{1}{\sigma}\left[i_t-E_t\left(\pi_{t+1}\right)-r_t^n\right]\end{aligned}$$

- Small open economy New Keynesian model

$$\begin{aligned}\pi_{H,t} &= \beta E_t\left(\pi_{H,t+1}\right)+\kappa_\alpha\tilde{y}_t\\ \tilde{y}_t &= E_t\left(\tilde{y}_{t+1}\right)-\frac{1}{\sigma_\alpha}\left[i_t-E_t\left(\pi_{H,t+1}\right)-r_t^n\right]\end{aligned}$$

- Equilbrium dynamics identical *but* in the open economy
  - Degree of openness affects slope of Phillips curve and sensitivity of output gap to interest rate changes
  - Natural rate of interest depends on expected world output growth
  - Distinction between domestic prices and CPI

Responses to shocks

- Given identical equilibrium dynamics, responses to shocks are qualitively identical to those in the closed economy
- Expected world growth new source of shock to natural rate of interest (in addition to technology shock)
- Size of the effect of the shock relative to closed economy depends on a number of parameters
- If η and γ are high (high subsitutability of goods produced in different countries), we have

$$\begin{aligned}
\omega &= \sigma\gamma + (1-\alpha)(\sigma\eta - 1) > 1 \\
\Theta &= \omega - 1 > 0 \\
\frac{\partial \sigma_\alpha}{\partial \alpha} &< 0 \\
\sigma_\alpha &< \sigma \\
\kappa_\alpha &< \kappa
\end{aligned}$$

Responses to shocks

- E.g. a positive shock to monetary policy with policy rule

$$\begin{aligned} i_t &= \rho + \phi_\pi \pi_{H,t} + \phi_\pi \tilde{y}_t + v_t \\ v_t &= \rho_v v_{t-1} + \varepsilon_t^v \end{aligned}$$

  where $\kappa_\alpha (\phi_\pi - 1) + (1 - \beta) \phi_y > 0$ to ensure determinacy
- Persistent decline in output and domestic inflation
- Declines are larger the smaller the economy *if* the international substitutability is sufficiently high
- International bookkeeping relations gives behaviour of open-economy variables
  - The terms of trade improves/falls (so that CPI inflation is sure to fall too)
  - The exchange rate appreciates (overshooting unless shock very persistent)

Optimal monetary policy

- In closed economy setting we had employment subsidy to focus attention on sticky price distortion
- Imperfect substitutability between domestic and foreign goods and sticky prices mean that central bank may want to influence terms of trade to favour domestic residents (Corsetti and Pesenti QJE 2001)
- For the special parameter configuration $\sigma = \eta = \gamma = 1$, the employment subsidy can be set to eliminate this distortion too resulting in an efficient flexible-price allocation
- Result: optimal monetary policy sets domestic inflation to zero (strict domestic inflation targeting) and optimal allocation attained

$$\tilde{y}_t = \pi_{H,t} = 0$$

- Implementation:

$$i_t = r_t^n + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t$$

Optimal monetary policy

- Implication: Under strict domestic inflation targeting (DIT), real variables behave as under flexible prices
- Since domestic prices are fully stabilised we have

$$\begin{aligned} e_t^{DIT} &= s_t^n - p_t^* \\ p_t^{DIT} &= \alpha \left(e_t^{DIT} + p_t^* \right) = \alpha s_t^n \end{aligned}$$

where

$$s_t^n = \sigma_\alpha \left(\frac{v - \mu}{\sigma_\alpha + \varphi} + \frac{1 + \varphi}{\sigma_\alpha + \varphi} a_t - \frac{\sigma + \varphi}{\sigma_\alpha + \varphi} y_t^* \right)$$

- Statistical properties of natural terms of trade inherited by CPI and nominal exchange rate
- Large and persistent fluctuations in nominal exchange rate and CPI inflation may be consequence of optimal policy!

Simple monetary policy rules

- Welfare loss function

$$\frac{1-\alpha}{2}\left[\frac{\varepsilon}{\lambda} VAR\left(\pi_{H,t}\right)+(1+\varphi)\, VAR\left(\tilde{y}_t\right)\right]$$

- Domestic inflation-based Taylor rule (DITR)

$$i_t = \rho + \phi_\pi \pi_{H,t}$$

- CPI inflation-based Taylor rule (CITR)

$$i_t = \rho + \phi_\pi \pi_t$$

- Exchange rate peg (PEG) imposes

$$e_t = 0$$

Simple monetary policy rules

- Calibration

$$
\begin{aligned}
\beta &= 0.99 \\
\sigma &= \eta = \gamma = 1 \\
\varphi &= 3 \\
\alpha &= 0.4 \\
\varepsilon &= 6 \\
\theta &= 0.75 \\
\phi_\pi &= 1.5 \\
(\rho_a, \sigma_a) &= (0.66, 0.0071) \\
\left(\rho_{y^*}, \sigma_{y^*}\right) &= (0.86, 0.0078)
\end{aligned}
$$

Simple monetary policy rules

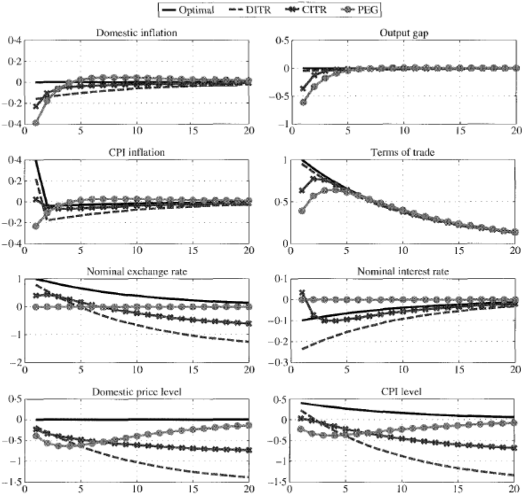


FIGURE 1

Impulse responses to a domestic productivity shock under alternative policy rules

Simple monetary policy rules

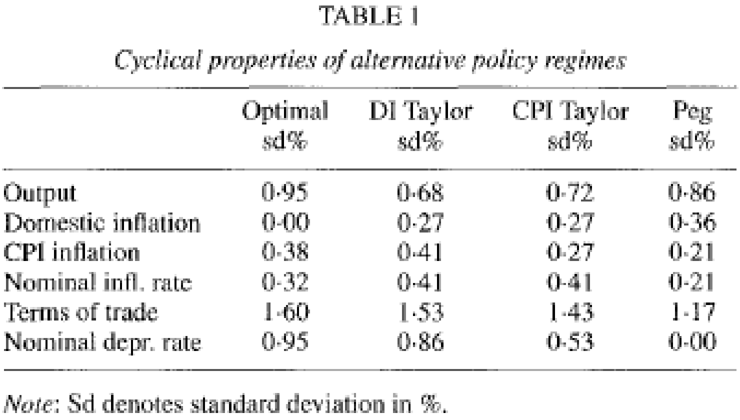


TABLE 1

Cyclical properties of alternative policy regimes

	Optimal sd%	DI Taylor sd%	CPI Taylor sd%	Peg sd%
Output	0·95	0·68	0·72	0·86
Domestic inflation	0·00	0·27	0·27	0·36
CPI inflation	0·38	0·41	0·27	0·21
Nominal infl. rate	0·32	0·41	0·41	0·21
Terms of trade	1·60	1·53	1·43	1·17
Nominal depr. rate	0·95	0·86	0·53	0·00

Note: Sd denotes standard deviation in %.

Simple monetary policy rules

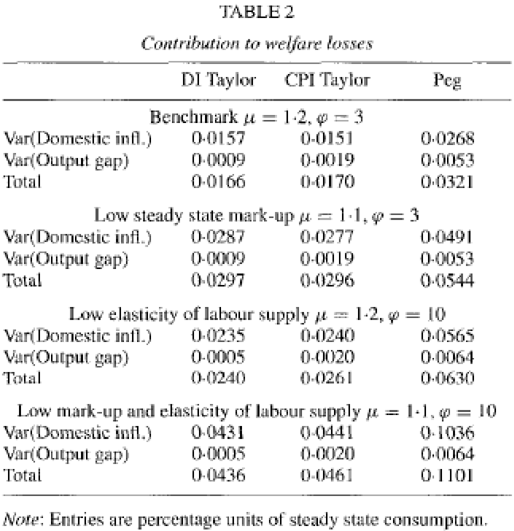


TABLE 2

Contribution to welfare losses

	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1{\cdot}2, \varphi = 3$			
Var(Domestic infl.)	0·0157	0·0151	0·0268
Var(Output gap)	0·0009	0·0019	0·0053
Total	0·0166	0·0170	0·0321
Low steady state mark-up $\mu = 1{\cdot}1, \varphi = 3$			
Var(Domestic infl.)	0·0287	0·0277	0·0491
Var(Output gap)	0·0009	0·0019	0·0053
Total	0·0297	0·0296	0·0544
Low elasticity of labour supply $\mu = 1{\cdot}2, \varphi = 10$			
Var(Domestic infl.)	0·0235	0·0240	0·0565
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0240	0·0261	0·0630
Low mark-up and elasticity of labour supply $\mu = 1{\cdot}1, \varphi = 10$			
Var(Domestic infl.)	0·0431	0·0441	0·1036
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0436	0·0461	0·1101

Note: Entries are percentage units of steady state consumption.

Simple monetary policy rules

- Optimal policy stabilises domestic prices and output (in loss function), but implies high volatility of terms of trade and hence the exchange rate (not in loss function)
- A peg does the reverse!
- Taylor rules are inbetween
- In terms of welfare losses, simple rules ranked
    - Domestic Taylor (DITR)
    - CPI Taylor (CITR)
    - Fixed exchange rate (PEG)

Simple monetary policy rules

- De Paoli JIE 2009 generalises (and hence complicates) analysis
  - Presents model in two-country set-up, which has both closed economy and small open-economy versions as special cases
  - Includes 'cost-push shock' (so strict inflation targeting no longer optimal)
  - Derives loss function in general case
- Loss function quadratic in domestic inflation, the output gap *and* the real exhange rate
- Optimal plan is quite complicated targeting rule in these variables
- Domestic inflation targeting only optimal under particular parameterisation considered by Galí and Monacelli RES 2005 (and only when no cost-push shocks)
- Intuition: terms of trade effect

Simple monetary policy rules

- De Paoli JIE 2009 (θ is the international elasticity of subsitution, ρ^{-1} is the intertemporal elasticity of substitution)

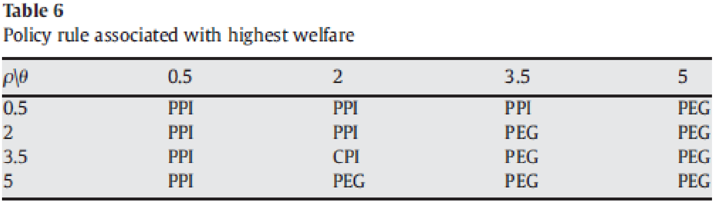


Table 6
Policy rule associated with highest welfare

$\rho \backslash \theta$	0.5	2	3.5	5
0.5	PPI	PPI	PPI	PEG
2	PPI	PPI	PEG	PEG
3.5	PPI	CPI	PEG	PEG
5	PPI	PEG	PEG	PEG

Main lessons

- This was an introduction to New Keynesian model
- Larger versions of models developed to better fit the data
- Identification and analysis of various distortions and frictions (including policy responses)
- Estimated often using Bayesian approach
- Main insights
  - Importance of expectations
  - Importance of natural levels of output and the interest rate (virtual flexible price equilibrium)

Extentions

- State-dependent pricing (alternatives to Calvo scheme)
- Labour market frictions and unemployment
- Imperfect inflation and learning
- Endogenous capital accumulation
- Financial market imperfections
- Zero bound on noimnal interest rates